(11 pts) 1. If $f(x,y,z) = \ln(x^2 + y^2 + 1) + y + 6z^2$ find

a) $\nabla f(1,1,0)$,

b) the direction of maximum change of f at $(1,1,0)$

c) $\dfrac{df}{ds}$ at $(1,1,0)$ in the $\dfrac{i}{3} + \dfrac{2j}{3} - \dfrac{2k}{3}$ direction.

(11 pts) 2. Find the volume of the solid over the triangle bounded by $y = 0$, $y = x$, and $x = 1$ under $z = 3 - x - y$.

(11 pts) 3. Find all maxima, minima, and points of inflection for $f(x,y) = 4xy - x^3 - y^3$.

(11 pts) 4. If $f(x,y,z) = x^2y + yz - z$ subject to $x^2 + y^2 + z^2 = 6$ find $\left(\dfrac{\partial f}{\partial x}\right)_z$ at $(x,y,z) = (2,1,1)$.

(11 pts) 5. Find the tangent plane and normal line to $x^2 + y^2 + z^2 = 30$ at $(1,2,5)$.

(11 pts) 6. The volume of a cone is $\dfrac{\pi r^2 h}{3}$. If the volume is computed for $r = 4$ and $h = 4$, estimate the error if r, in fact, is 4.2 and h, in fact, equals 3.9.

(11 pts) 7. Find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the surface $x^2 + y^2 + z^2 = 25$.

(12 pts) 8. If $f(x,y) = \dfrac{1}{1 - x - y}$

a) find the linear approximation, $\ell(x,y)$ near $(0,0)$,

b) find the quadratic approximation, $q(x,y)$ near $(0,0)$,

c) estimate the error $|f(x,y) - \ell(x,y)|$ if $|x| < 10^{-2}$ and $|y| < 10^{-2}$.
HINT: $|x| < 10^{-2}$ and $|y| < 10^{-2}$ implies $1 - x - y \geq .98$.

Do EITHER 9) or 10) or 11). INDICATE YOUR CHOICE:

(11 pts) 9. If $f(x,y,z)$ has a maximum at P, a point on the surface $g(x,y,z) = 10$, show $\nabla f(p) = \lambda \nabla g(p)$.

(11 pts) 10. If a particle moves 10^{-2} units along the helm $x = 3\cos t$, $y = \sin t$, $z = 4t$ from $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, \pi\right)$ towards $(0, 3, 2\pi)$, t goes from $\frac{\pi}{4}$ towards $\frac{\pi}{2}$, and $f(x, y, z) = x^2 + y^2 + z$ estimate ΔF.

(11 pts) 11. a) If $f, f_x, f_y, f_{xx}, f f_{xy}, f_{yy}$ are continuous state the Taylor Formula of order 2 with (x_0, y_0) as a starting point.
I.e. $f(x, y) = f(x_0, , y_0) + ?$

b) Use the formula of part a) to show $|f(x, y) - \ell(x, y)| \leq \frac{M}{2}[|x - x_0| + |y - y_0|]^2$ where $M =$ maximum of $|f_{xx}|$, $|f_{xy}|$, $|f_{yy}|$.

MATHEMATICS 182 TEST 3

(15 pts) 1) Set up integrals **but do not evaluate them** for the mass of the solid between $z = \sqrt{x^2+y^2}$ and $x^2+y^2+z^2 = 9$ if the density $\delta = x$ in

A) Rectangular coordinates,

B) Spherical coordinates,

C) Cylindrical coordinates.

(14 pts) 2) Change the integral

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dydx$$

into polar coordinates and evaluate it.

(14 pts) 3) Evaluate $\int_C f ds$ if $f = xy^2$ and C is the line connecting $(0,0,0)$ to $(1,2,-1)$.

(14 pts) 4) Find the mass of the volume above $z = y^2$, below $z = 4$, and between $x = 0$ and $x = 1$ if the density $\delta = x$.

(12 pts) 5) Find the average distance from $(0,0,0)$ to a point (x,y,z) belonging to the set

$$R = \{(x,y,z)|x^2+y^2+z^2 \leq 4).$$

(11 pts) 6) Find the work done by the force $\vec{F} = yi + xj + x^2k$ over the curve $x = \cos t,\ y = \sin t,\ z = t,\ 0 \leq t \leq 2\pi$.

(6 pts) 7) Let $R = \{(u,v)1 \leq u \leq 1.01,\ 1 \leq v \leq 1.01\}$ be a set in the $u-v$ plane. Let $x = uv^2$ $y = u^2v + uv$ be a map from the $u-v$ plane into the $x-y$ plane. If $\overline{R}$ is the image of R under the given map what is the approximate area of $\overline{R}$.

(14 pts) 8) Let R be the region in the $x-y$ plane bounded by $y = 0,\ y = x$, and $x + 2y = 2$ use the following steps to evaluate

$$\int_R (x+2y)e^{y-x}dA \text{ using}$$

the substitution $u = x + 2y$ $v = x - y$.

A) Sketch R and its image in the $u-v$ plane.

B) Find $\dfrac{\partial(x,y)}{\partial(u,v)}$.

C) Evaluate the integral.