182 TEST 3 -597 NAME _____

Don't do arithmetic

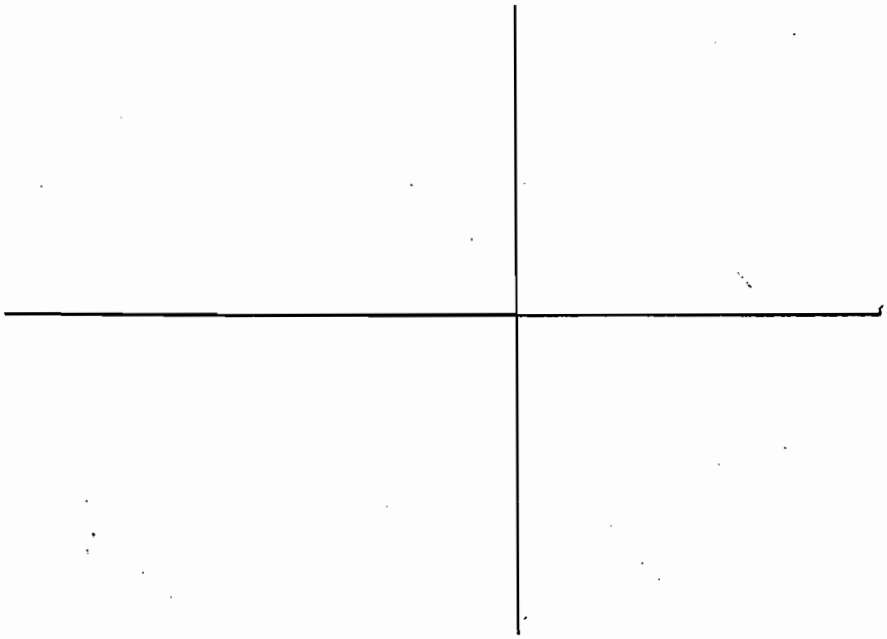
182-3-597.pdf

1. (a) Give the equation of a vector field $\overrightarrow{F(x,y)}$ such that [illegible] origin from $(x,y)$ and [illegible] toward the [illegible] 5 3

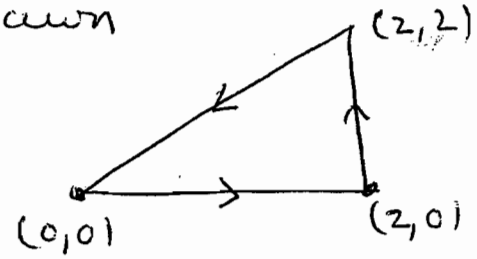
(b) Give the equation of any vector field $\overrightarrow{G(x,y)}$ such that $\overrightarrow{G(x,y)}$ is tangent to the circle around the origin containing $(x,y)$ and points in the clockwise direction. 5

2. Find $\int_C 5x^4y^6\,dx + (6x^5y^5 + e^y)\,dy$, where C is that part of the graph of $y = x^3 + 1$ which starts at $(1,2)$ and ends at $(2,9)$. 5

3. Sketch the graph of $3x^2 + 3y^2 - 2xy = 1$.
(a 45° counterclockwise rotation works)



4. (10) (a) Evaluate $\int_{L_1} y\,dx$, $\int_{L_2} y\,dx$, and $\int_{L_3} y\,dx$, where $L_1$, $L_2$, and $L_3$ are the sides of the triangle drawn




(5) (b) Evaluate $\oint y\,dx$ around the above triangle by using Green's Theorem

5. Find $\iint_P (2x-y)\,dx\,dy$, where P is the parallelogram bounded by $x+y=0$, $x+y=1$, $2x-y=1$, and $2x-y=3$; by changing variables

6. Let $S$ be that part of the plane $z = 2x + 3y + 1$ which lies directly above the unit square $0 \le x \le 1, 0 \le y \le 1$ of the $xy$ plane

6 (a) Find the area of $S$

7 (b) Find the flux across $S$ from the field $\vec{F}(x,y,z) = 3\vec{k}$ (up counts as +)

7 (c) Find the area of the perpindicular projection of $S$ onto the $xz$ plane. (The shadow on the $xz$ plane of $S$ if the sun's rays come in parallel to the $y$ axis from the $+y$ direction.

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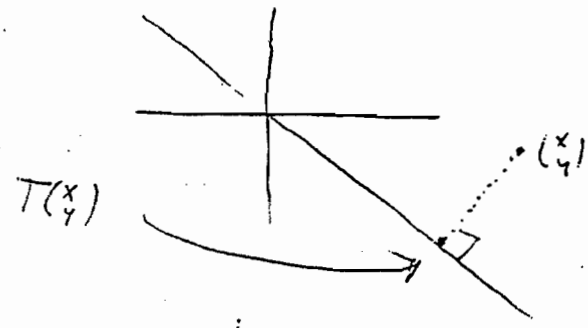
7. Let $T\begin{pmatrix}x\\y\end{pmatrix}$ be the perpindicular projection of $\begin{pmatrix}x\\y\end{pmatrix}$ onto the line $y=-x$ :

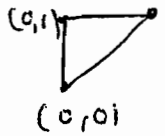



Let $S\begin{pmatrix}x\\y\end{pmatrix}$ be the perpindicular projection of $\begin{pmatrix}x\\y\end{pmatrix}$ onto the line $y=-x+1$.

One of $T$, $S$ can be represented by a matrix. Say which one can, and give the matrix. Briefly say why the other one can't.

5 8(a) Find the equation of a map $T(u,v)$ which maps a rectangle in the $uv$ plane onto a disc in the $xy$ plane

5 (b) Find the equation of a map $S(u,v)$ which maps the unit square $0 \le u \le 1$, $0 \le v \le 1$ of the $uv$ plane onto the parallelogram which has corners $(0,0)$, $(0,2)$, $(2,3)$, $(2,1)$.

10 9 Suppose $T(u,v) = (x(u,v), y(u,v))$ maps the unit square $0 \le u \le 1$, $0 \le v \le 1$, onto the triangle 

in the $xy$ plane. Suppose the Jacobian of this map $J(u,v) = \frac{\partial(x,y)}{\partial(u,v)}$ is a positive constant $k$. What is $k$? (Your answer will be a single number. Briefly explain your answer)