MATHEMATICS 182 TEST IIIA

1) $\mathbf{F} = (y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$.

2) Find the mass of a thin wire lying along the curve $\mathbf{r}(t) = \sqrt{2}\, t\mathbf{i} + \sqrt{2}\, t\mathbf{j} + (4 - t^2)\mathbf{k}$, $0 \leq t \leq 1$, if the density is $\delta = 3t$.

3) Find the mass of a thin plate covering the region outside the circle $r = 3$ and inside the circle $r = 6 \sin \theta$ if the plate's density function is $\delta(x, y) = 1/r$.

4) **Set up but do not evaluate integrals for the following:**

a) The solid bounded below by the hemisphere $\rho = 1$, $z \geq 0$, and above by the cardioid of revolution $\rho = 1 + \cos \phi$.

b) Find the volume of the region bounded above by the paraboloid $z = 9 - x^2 - y^2$, below by the xy–plane, and lying outside the cylinder $x^2 + y^2 = 1$.

c) Find the moment of inertia of a right circular cone of base radius a and height h about its axis. (Hint: Place the cone with its vertex at the origin and its axis along the z-axis.)

5) Find the circulation and flux of the fields $F = x\mathbf{i} + y\mathbf{j}\ x\mathbf{j}$ around and across the following curve: The ellipse $\mathbf{r}(t) = (\cos t)\mathbf{i} + (4 \sin t)\mathbf{j}$, $0 \leq t \leq 2\pi$.

6) $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$.

7) a) Solve the system $u = 2x - 3y$, $v = -x + y$ for x and y in terms of u and v. Then find the value of the Jacobian $\partial(x, y)/\partial(u, v)$.

b) Find the image under the transformation $u = 2x - 3y$, $v = -x + y$ of the parallelogram R in the xy-plane with boundaries $x = -3$, $x = 0$, $y = x$, and $y = x + 1$. Sketch the transformed region in the uv-plane.

c) Use the transformation and parallelogram R in a) to evaluate the integral

$$\iint_R (2x - y)\,dx\,dy.$$

MATHEMATICS 182 TEST 3

(10 pts) 1) Find I_z, the moment of inertia with respect to the z–axis of the volume between $z = x^2 + y^2$ and $z = 1$ if the density $\delta = z$.

(10 pts) 2) Change the integral

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x dy dx$$

to polar coordinates and evaluate it.

(10 pts) 3) Find the area outside $r = 1$ and inside $r = 1 + \cos\theta$.

(10 pts) 4) Evaluate the line integral of $f(x, y, z) = x + y + z$ along the line connecting $(1, 1, 1)$ to $(2, 3, -2)$.

(10 pts) 5) Find the work done by the force $\vec{F} = x^2 i + z^2 k$ over the curve

$$x = \cos t \qquad y = \sin t \qquad z = t,\ 0 \le t \le \pi.$$

(10 pts) 6) Find the volume above $z = y^2$ and below $z = 4$ between $x = 0$ and $x = 1$.

(30 pts) 7) Set up but do not evaluate integrals for the following.

a) The mass of the tetrahedron with corners $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ if the density $\delta = 2y$,

b) The center of mass of the plate bounded by the parabola $y^2 = 4x$ and the line $x + y = 4$ if the density $\delta = 1$,

c) The volume between the spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 1$ above the cone $\varphi = \pi/4$.

(10 pts) 8) a) Solve the system

$$u = x + 2y \qquad \text{and} \qquad v = x - y \text{ for}$$

$\overset{x}{\cancel{u}}$ and $\overset{y}{\cancel{v}}$. Find the Jacobian $J(u, v)$.

b) Sketch the region in the $x - y$ plane bounded by $y = 0$, $y = x$, and $x + 2y = 2$. What is the image in the $u - v$ plane?

c) Change $\displaystyle\int_0^{2/3}\int_y^{2-2y} (x + 2y)e^{y-x}\,dx\,dy$ into an integral over a domain in the $u - v$ plane. Do not evaluate the integral.