MATHEMATICS 182 FINAL EXAM

(20 pts) 1) Let $f(x, y, z) = x^2 \cos y + z^2$.

a) Find ∇f at $(1, 0, 1)$.

b) Find the directional derivative of f in the direction of $\vec{V} = 2k$.

c) In what direction does f change most rapidly?

(20 pts) 2) Given the points $P_1 = (1, 2, 1)$, $P_2(2, 2, 3)$, and $P_3 = (2, -1, 1)$ find

a) the equations of a line through P_1 and P_2,

b) the equation of a plane through P_1, P_2, and P_3.

(15 pts) 3) If $w = x^2 + y^2$, $x = r - s$, and $y = r + s$ find $\dfrac{\partial^2 w}{\partial r^2} + \dfrac{\partial^2 w}{\partial s^2}$

(25 pts) 4) a) Show that the vector field
$\vec{F} = (z + \cos y)i + [-x \sin y)j + (x + z^2)k$
is conservative.

b) Find f such that $\vec{F} = \nabla f$.

c) What is $\displaystyle\int_C \vec{F} \circ dR$ if C is the curve $x = t^3,\ y = \pi t^2,\ z = t,\ 0 \leq t \leq 1$.

(20 pts) 5) a) Find parametric equations for the cone
$3(x^2 + y^2) = z^2\ ,\ 0 \leq z \leq \sqrt{3}$

b) Find the surface area of the cone.

(20 pts) 6) a) Use the divergence theorem to evaluate $\int_S (\vec{V} \circ \vec{N}) d\sigma$ where $\vec{V} = 3xi - yj - zk$ and S is the sphere $x^2 + y^2 + z^2 = 9$.

b) Use Stoke's Theorem to evaluate $\int_S (\nabla \times \vec{V}) \circ \vec{N} d\sigma$ if $\vec{V} = x^2y^2i$ and S is the <u>upper half</u> of $x^2 + y^2 + z^2 = 9$.

(20 pts) 7) Find the mass of the paraboloid $z = 1 - x^2 - y^2,\ z \geq 0$, if the density $\int = x^2$.

(25 pts) 8) If $x = \cos t,\ y = \sin t$, and $z = 4t$ find

a) the unit tangent vector as a function of t,

b) $\dfrac{ds}{dt}$ as a function of t,

c) the curvature $\kappa = \left|\dfrac{d\vec{T}}{ds}\right|$ as a function of t.

(20 pts) 9) a) Find all critical points of
$f(x, y) = 9x^3 + y^3/3 - 4xy$

b) Decide which of the critical points are maxima, minima, at saddle points.

(15 pts) 10) a) If a twice differentiable vector field $\vec{V} = v_, i + v_2 j + v_3 k$ is the gradient of a scalar function f, $\vec{V} = \nabla f$, show $\nabla \times \vec{V} = 0$.

b) If $\int_C \vec{V} \cdot d\vec{R}$ depends on the endpoints only and if $\vec{V}$ is twice differentiable show $\vec{V} = \nabla f$ for some scalar function f.