182FE-S03

MATHEMATICS 182 FINAL EXAM

1) Given the points $P_1 = (1,1,1)$, $P_2 = (2,3,1)$, and $P_3 = (2,-1,2)$ find

   a) The equation of the line through P_1 and P_2,

   b) The equation of the plane through P_1, P_2, and P_3.

   c) The intersection point of the line of part a) with the $y - z$ plane.

2) a) If z is defined as a a function of x and y by the equation $xz + yz^3 - 2xy = 0$ find $\dfrac{\partial z}{\partial x}$ at $(1,1,1)$.

   b) If $z = \ln(w)$ and $w = (\sqrt{v+3})(\tan^{-1} u)$ find $\dfrac{\partial z}{\partial v}$ and the $\dfrac{\partial z}{\partial r}$ when $u = 1$ and $v = -2$.

3) a) Show that the vector field $\vec{F} = (z^2 + \cos y)i + (-x \sin y)j + \left(2xz + \cos \dfrac{\pi z}{2}\right)k$ is conservative.

   b) Find f such that $\vec{F} = \nabla f$.

   c) What is $\displaystyle\int_C \vec{F} \cdot dR$ if C is the curve $x = \cos t \quad y = \dfrac{t}{\pi} \quad z = \sin t,\ 0 \le t \le 4\pi$?

4) If a curve is parametrized by $x = \cos t \quad y = \sin t \quad z = 4t,\ -\infty < t < \infty$, find

   a) The unit tangent vector, $\vec{T}$, as a function of t,

   b) $\dfrac{ds}{dt}$ as a function of t,

   c) $K = \left|\dfrac{d\overrightarrow{T}}{ds}\right|$ as a function of t.

   d) $\vec{N}$ as a function of t.

   e) $\vec{B}$ as a function of t.

5) If $f(x,y,z) = x^2y + xye^z$ find

   a) ∇f at $(1,2,0)$,

   b) the direction in which f changes most rapidly at $(1,2,0)$,

   c) the tangent plane and normal line to $x^2y + xye^z = 4$ at $(1,2,0)$.

6) a) Find all critical points of $9x^3 + \frac{y^3}{3} - 4xy$.

b) Which of the critical points are maxima, minima, saddle points.

7) If $f(x, y) = y \sin x$ find

a) the values of f_{xx}, f_{xy}, f_{yy} at $(0, 0)$

b) the quadratic approximation of f.

8) a) Parametrize the part of the spherical surface $x^2 + y^2 + z^2 = 9$ over the cone $z = \sqrt{x^2 + y^2}$.

b) What is the surface area of the surface of a)

9) Set up but do not evaluate integrals for the following:

a) The area inside the cordiod $r = 2(1 + \sin\theta)$ and outside the circle $r = 1$,

b) The mass of the tetrahedron with corners $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$ if the density $\delta = xy$.

c) The volume between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$ inside the sphere $x^2 + y^2 + z^2 = 9$.

10) a) Use Stoke's theorem to evaluate $\int_S (\nabla \times \vec{F}) \cdot \vec{N} d\sigma$ if $\vec{F} = 2zi + 3xj + 5yk$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ above the $x - y$ plane.

b) Use Green's Theorem to find the area inside the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$