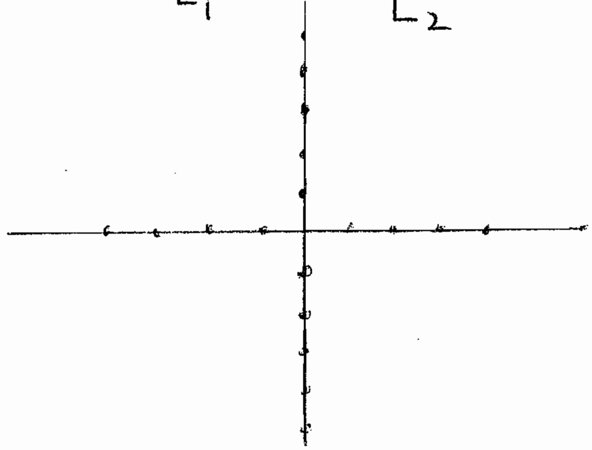
Davis Spring 97 182 Final Exam Name ____________

25 points / problem Don't do arithmetic

1. (a) Find $\oint_C y\,dx$, where C is the circle of radius 2 around $(0,0)$, by parametrizing C and calculating an integral.

(b) Find $\int_C y\,dx$, where C is the circle of radius 2 around $(0,0)$, by using Green's theorem.

(c) Draw line segments L_1 and L_2 below such that L_1 and L_2 have length 4, L_1 is perpendicular to L_2, and $\int_{L_1} y\,dx = \int_{L_2} y\,dx$



2. Set up triple integrals (or sums of triple integrals) which give the volume of that part of the sphere of radius 5 about the origin which lies within $\sqrt{5}$ of the z axis and above the x-y plane. (That is, the intersection of a half-sphere and a cylinder).

(a) In rectangular coordinates

(b) In cylindrical coordinates

(c) In spherical coordinates

3. The curve $(2\cos t, t, 2\sin t)$, $0 \le t \le 2\pi$, parametrizes a curve

(a) Sketch and describe with words this curve

(b) Find the length of this curve

(c) Give, in parametric form, the equation of the tangent line to this curve at $(-2, \pi, 0)$

(D) For times t close to π, $\vec{r}(t) = (2\cos t, t, 2\sin t)$ almost lies in a plane. Give its equation.

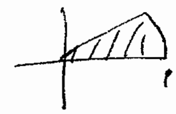
4. Let $f(x,y) = x^3 + xy$.

(a) Give the equation of the tangent plane to the graph of f at $(1,1,2)$.

(b) Give a vector $\vec{v}$ such that the directional derivative of f in the direction of $\vec{v}$ at $(1,1)$ is greatest, and a vector $\vec{w}$ such that the directional derivative of f in the direction of $\vec{w}$ is zero, at $(1,1)$.

(c) Find the Taylor expansion of f around $(0,0)$, out through the quadratic terms.

5. Find the mass and the center of mass of a wedge consisting of that part of the plane in the first quadrant bounded by the unit circle, the x-axis, and the line $y = x$, if the mass density at (x, y) equals $\sqrt{x^2+y^2}$.



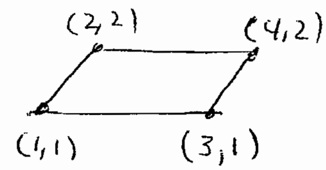
6. (a) The map $T(u,v) = (u\cos v, 2u, u\sin v)$, $0 \le u \le 3$, $0 \le v \le 2\pi$ parametrizes what surface (sketch)

(b) Find a normal vector to this surface at the point $(1,2,0)$.

(c) Find the flux (away from y axis is +) across this surface from $\vec{F}(x,y,z) = 7\vec{k}$

7. (a) Use Jacobians to find $\iint_P (x+y)\,dx\,dy$,

where P is the parallelogram



(b) Find a map $T(u,v)$ which maps a rectangle in the u-v plane onto the parallelogram above.