MA271, Fall 2000
Midterm 1

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Sep 21, 2000

- This booklet has SIX QUESTIONS, each worth 10 points. You have 50 minutes to do this test. **Plan your time well. Read the questions carefully.**

- This test is closed books and closed notes.

- Any calculator is allowed. But it cannot be used for justifying answers.

- In order to get full credit, your answers need to be **correct** and **simplified**. You need to explain in a **comprehensible** way how you obtained them.

- Both sides of the paper may be used. **Indicate** which problem you work on whenever you use a backside.

Question 1.
Consider a cube with side length $a = 1$ and corners as indicated in the picture.
Find the area of the triangle with corners A, B and C.

Question 2.
The curves
$$y = x^2 - 1$$
and
$$y = -x^2 + 1$$
intersect in the points $(1,0)$ and $(-1,0)$. Find the angle between the curves in the point $(1,0)$.

Question 3.

Find $\lim_{n \to \infty} n \cdot (1 - \cos(\frac{1}{n}))$.

Question 4.
Determine the convergence/divergence of the following series and clearly indicate your reasoning.

(a) $\displaystyle\sum_{2}^{\infty} \frac{1}{n\cdot(\ln(n))^{1+a}}$ where $a < 0$.

(b) $\displaystyle\sum_{1}^{\infty} \frac{2n+3^n}{n^2\cdot3^n}$.

(c) $\displaystyle\sum_{1}^{\infty} \left(\frac{n-2}{n}\right)^{2n}$.

Question 5.

(a)   For the series $\sum_1^\infty \frac{(x-3)^n}{\sqrt{(n!)}}$ find the radius of convergence and the region of convergence.

(b)   Also, find the value of the series for $x = 2$ with an error smaller than $\frac{1}{100}$.

Question 6.

(a) Find the radius of convergence of the power series $f(x) = \sum_{1}^{\infty} \left(\frac{x^2+1}{9} \right)^n$.

(b) Express f as a function of x **without using the sum notation**.

(c) Find $f'(2)$.