MA 271 **EXAM 2** **FALL 2004** **Name** ____________________

1. Find the limit of each convergent sequence.

   (a) $a_n = \dfrac{1 - 10n + n^2}{3n^2 - 8}$.

   (b) $a_n = n - \sqrt{n^2 - 2n}$.

   (c) $a_n = \dfrac{\sin^2(\frac{1}{n})}{1 - \cos(\frac{3}{n})}$.

2. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{2^n}$.

3. Determine which of the series converge absolutely, converge conditionally, or diverge. Give reason for your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \tan^{-1} n}{n^2+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n 3^n}{n^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{(2n)!}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

4. Find the Taylor polynomial of $(1 + x^2)\cos x$ of order 7 at $x = 0$.

5. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2x-3)^n}{n5^n}.$$

6. The approximation $\cos x \sim 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!}$ is used. Determine the smallest n needed to estimate $\cos(0.1)$ with an error of less than 10^{-12}.

7. Find a series solution for $y' - xy = 0,\ y(0) = 1$.

8. $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x-y}{x^2-y^2}$.

9. Compute $xf_x + yf_y + zf_z$. $f(x,y,z) = xy + yz + zx$.

10. Find $\frac{dw}{dt}$ at $t = 0$ if $w = \sin(xy + \pi)$, $x = e^t$, and $y = \ln(t+1)$.