F04

MATHEMATICS 271 TEST NUMBER 3

(21 pts) (1) a) Find the value of

$$\frac{\partial u}{\partial x} \text{ at } (x, y, z) = (2, 1, -1) \text{ if}$$

$$u = \frac{p-q}{r}, \quad p = x + y + z,$$

$$q = x - y + z, \quad \text{and} \quad r = x + y - z.$$

b) Find the linearization of

$$f(x, y) = e^x \cos y \text{ at } (0, 0).$$

c) Estimate the error if $f(x, y) = e^x \cos y$ is replaced by its linearization at $(0, 0)$ assuming $|x| < .01$ and $|y| < .01$.

(28 pts) (2) If $f(x, y, z) = x^2 + y^2 - 2z^2 + z\ln(x)$ find

a) ∇f at $(1, 1, 1)$,

b) the direction in which f changes most rapidly at $(1, 1, 1)$,

c) the tangent plane to $x^2 + y^2 - 2z^2 + z\ln x = 0$ at $(1, 1, 1)$,

d) the normal line to $x^2 + y^2 - 2z^2 + z\ln x = 0$ at $(1, 1, 1)$.

(30 pts) (3) a) Find the mass of a thin plate bounded by $y = x^2$ and $y = x$ if the density $\delta = 2xy$.

b) Evaluate

$$\int_0^2 \int_x^2 2y^2 \sin(xy)\,dy\,dx$$

by changing the order of integration.

c) Describe the average value of $f(x, y, z) = xyz$ over the region bounded by $z = x^2 + y^2$ and $4y + z = 12$ *as the ratio of two integrals.* **Do not evaluate the integrals.**

d) Set up a triple integral, **but do not evaluate it**, for the volume bounded by $\rho = 1$ and $\varphi = \pi/6$.

(21 pts) (4) a) Find all maxima, minima, and saddle points for

$$f(x,y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$$

b) Find the absolute maximum and the absolute minimum of

$$f(x,y) = x^2 + 2y^2 - x \qquad \text{on}$$
$$\{(x,y) | x^2 + y^2 \leq 1\}.$$

c) Find the point on

$$x^2yz = 1 \qquad \text{nearest to } (0,0,0).$$