MA 271 FINAL EXAM FALL 2004 Name ____________________

1. Let f be a differentiable function of one variable and $u(x,y,z) = (x^2+y^2+z^2)\, f(\frac{xyz}{x^2+y^2+z^2})$. Determine G and H such that

$$xu_x + yu_y + zu_z = G(x,y,z)\, f(\frac{xyz}{x^2+y^2+z^2}) + H(x,y,z)\, f'(\frac{xyz}{x^2+y^2+z^2}).$$

2. Find an equation for the tangent plane to the surface $x^2 - 3y^2 + z^2 = 1$ at $(-3, 2, -2)$.

3. Find an equation for the tangent line to the intersection of $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$ and $2x + 3y + z = -1$ at $(1, -2, 3)$.

4. Classify the critical points of the function $f(x,y) = x^3 - y^3 - 3xy$.

5. Find the maximum value of xyz subject to the constrain $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

6. Evaluate

$$\int_0^1 \int_{y^{\frac{1}{3}}}^1 \sin(x^4)\, dx\, dy.$$

7. Set up a triple integral for the volume of the solid bounded by the paraboloid $z = 9 - x^2 - y^2$ and $z = 0$.

8. Find the area of the image of the rectangle $[-1, 1] \times [0, 1]$ under the transformation $T(u, v) = (2u, 3u + v^2)$.

9. Let C be the curve given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, $0 \leq t \leq 1$. Evaluate the line integral

$$\int_C (y - z)dx + (z - x)dy + (x - y)dz.$$

10. Find the work done by the force $\mathbb{F} = \langle y \sin xy, x \sin xy \rangle$ along the line segment from the origin to $(1, \frac{\pi}{2})$.

11. Find a potential function f for the field

$$\mathbb{F} = \langle y + z,\ z + x,\ x + y \rangle.$$

12. Let C be the circle $(x - 2)^2 + (y - 3)^2 = 4$. Evaluate the line integral

$$\oint_C (6y + x)\, dx + (y + 2x)\, dy.$$

13. Let S be the portion of the parabolid $z = \frac{1}{4} + x^2 + y^2$, $z \leq \frac{5}{4}$. Compute

$$\iint_S z d\sigma.$$

14. Let S be the surface $x^2 + y^2 + z^2 = 4$, $z \geq 0$ with upward normal and $\mathbb{F} = \langle -y + z \sin x,\ x + \sin z,\ -z + x \sin y \rangle$. Compute

$$\iint_S (\nabla \times \mathbb{F}) \cdot \mathbf{n} d\sigma.$$

15. Let $\mathbb{F} = \langle x + yz,\ y + zx,\ z + xy \rangle$ and S be boundary of the tetrahedron in the first octant bounded by the coordinate planes and $x + y + z = 1$ with outward normal. Compute

$$\iint_S \mathbb{F} \cdot \mathbf{n} d\sigma.$$