Solutions to the Practice Questions for the Final Exam
MA 153

1.

$$\frac{\frac{15}{5}}{\frac{1}{2}} = \frac{3}{\frac{1}{2}} = \frac{3}{1} \cdot \frac{2}{1} = 6 \qquad\qquad \textbf{D}$$

2.

$$16x^2 - 4y^8 = 4(4x^2 - y^8) = 4\left[(2x)^2 - \left(y^4\right)^2\right]$$

$$= 4\left(2x + y^4\right)\left(2x - y^4\right)$$

$\textbf{C}$

3.

$$\left(\frac{4a^4b^8}{c^{-2}}\right)^{-\frac{1}{2}} = \left(4a^4b^8c^2\right)^{-\frac{1}{2}} = \left(\frac{1}{4a^4b^8c^2}\right)^{\frac{1}{2}}$$

$\textbf{A}$

$$= \frac{\sqrt{1}}{\sqrt{4a^4b^8c^2}} = \frac{1}{2a^2b^4c}$$

4.

$$\frac{3x}{3x+1} - \frac{x}{x-2} = \frac{3x(x-2)}{(3x+1)(x-2)} - \frac{x(3x+1)}{(3x+1)(x-2)}$$

$\textbf{C}$

$$= \frac{3x^2 - 6x - 3x^2 - x}{(3x+1)(x-2)} = \frac{-7x}{(3x+1)(x-2)}$$

5.

$$\frac{\frac{x-2}{(x+1)(x-3)}}{\frac{(x-2)(x+1)}{(x+3)(x-3)}}$$

$\textbf{B}$

$$= \frac{x-2}{(x+1)(x-3)} \cdot \frac{(x+3)(x-3)}{(x-2)(x+1)} = \frac{x+3}{(x+1)^2}$$

6.

let t = time second person working alone

$$\frac{1}{6}+\frac{1}{t}=\frac{1}{4} \qquad \text{or} \qquad \frac{1}{6}(4)+\frac{1}{t}(4)=1$$

$$12t\left[\frac{1}{6}+\frac{1}{t}\right]=\left[\frac{1}{4}\right]12t \qquad\qquad 3t\left[\frac{2}{3}+\frac{4}{t}\right]=[1]3t \qquad\qquad \mathbf{E}$$

$$2t+12=3t \qquad\qquad\qquad 2t+12=3t$$

$$t=12 \qquad\qquad\qquad\qquad t=12$$

7. $$\frac{xy^{-1}}{(x+y)^{-1}}=\frac{\dfrac{x}{y}}{\dfrac{1}{(x+y)}}=\frac{x}{y}\cdot\frac{(x+y)}{1}=\frac{x(x+y)}{y} \qquad\qquad \mathbf{A}$$

8.

$$\frac{\sqrt{3}}{2+\sqrt{3}}\cdot\frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\mathbf{C}$$

$$=\frac{2\sqrt{3}-\left(\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2}=\frac{2\sqrt{3}-3}{4-3}=2\sqrt{3}-3$$

9.

let x = first positive integer $(x<y)$

$$\begin{cases} y=x+1 \\ \\ y^2-x^2=145 \end{cases} \Rightarrow \text{substitute eq. (1) into eq.(2) for y :}$$

$$(x+1)^2-x^2=145$$

$$x^2+2x+1-x^2=145 \qquad\qquad \mathbf{B}$$

$$2x+1=145$$

$$2x=144$$

$$x=72$$

10.

$$A = P(1 + rt)$$

$$A = P + Prt$$

$$A - P = Prt$$ **E**

$$\frac{A - P}{Pr} = t$$

11.   let t = # hours truck has been traveling
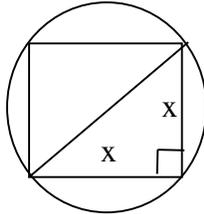
$$40t = 55(t - 1)$$

$$40t = 55t - 55$$

$$55 = 15t$$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t – 1)

$$t = \frac{55}{15} = \frac{11}{3} \text{ hours, so distance is } 40\left(\frac{11}{3}\right) = \frac{440}{3} \text{ miles}$$ **A**

12.

Let A = area of circle

Area of circle $\Rightarrow A(r) = \pi r^2$

Diameter (d) of circle $\Rightarrow x^2 + x^2 = d^2$

$$2x^2 = d^2$$

$$d = \pm\sqrt{2x^2}$$

$$d = x\sqrt{2}$$ **A**

Radius (r) of circle $\Rightarrow r = \frac{x\sqrt{2}}{2}$

So, $A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$

$$= \frac{\pi x^2}{2} \text{ or } \frac{\pi}{2}x^2$$

3

13.

$$\frac{4}{2p-3} + \frac{10}{4p^2-9} = \frac{1}{2p+3} \qquad \left(Domain: p \neq \pm\frac{3}{2}\right)$$

$$(2p+3)(2p-3)\left[\frac{4}{2p-3} + \frac{10}{(2p+3)(2p-3)}\right] = \left[\frac{1}{2p+3}\right](2p+3)(2p-3)$$

$$4(2p+3) + 10 = 2p-3 \qquad\qquad\qquad\qquad\qquad \mathbf{D}$$

$$8p + 12 + 10 = 2p-3$$

$$6p = -25$$

$$p = -\frac{25}{6}$$

14.

let $x = $ # ml of the 50% solution
let $y = $ total # of ml
$$\begin{cases} x + 40 = y \\ x(.50) + 40(.20) = y(.25) \end{cases} \qquad\qquad \mathbf{B}$$
$$x(.50) + 8 = (x+40)(.25)$$
$$.50x + 8 = .25x + 10$$
$$.25x = 2$$
$$x = 8 \text{ ml}$$

15.

$$x = \sqrt{14+5x} \qquad\qquad \text{Check}:$$

$$(x)^2 = \left(\sqrt{14+5x}\right)^2 \qquad\qquad \text{If } x = 7:$$

$$x^2 = 14 + 5x \qquad\qquad\qquad 7 = \sqrt{14+35}$$

$$x^2 - 5x - 14 = 0 \qquad\qquad\qquad 7 = \sqrt{49} \Rightarrow \text{ yes} \qquad\qquad \mathbf{E} \ \left[x = 7\right]$$

$$(x-7)(x+2) = 0 \qquad\qquad \text{If } x = -2:$$

$$x = 7, \qquad x = -2 \qquad\qquad\qquad -2 = \sqrt{14-10}$$

$$-2 = \sqrt{4} \Rightarrow \text{no}$$

16

$$m^4 - m^2 - 6 = 0$$

$$\left(m^2 - 3\right)\left(m^2 + 2\right) = 0$$

$$m^2 - 3 = 0, \quad m^2 + 2 = 0$$

D

$$m^2 = 3, \quad m^2 = -2$$

$$m = \pm\sqrt{3}, \quad m = \pm\sqrt{-2}$$

$$m = \pm\, i\,\sqrt{2}$$

17.

$$3x - 2 > 6x + 1$$

$$-3x > 3$$

A

$$x < -1$$

$$(-\infty,\ -1)$$

18.

$$|6 - 2x| \le 3$$

$$-3 \le\ 6 - 2x\ \le 3$$

$$-9 \le -2x \le -3$$

$$\frac{9}{2} \ge x \ge \frac{3}{2}$$

C

$$\frac{3}{2} \le x \le \frac{9}{2}$$

19.

$$2x^2 - 4x + k = 0$$

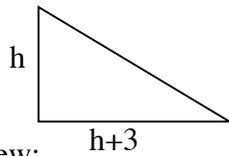$$x = \frac{4 \pm \sqrt{16 - 4(2)(k)}}{2(2)} = \frac{4 \pm \sqrt{16 - 8k}}{4}$$

$$\text{Need}: \quad 16 - 8k \ge 0$$
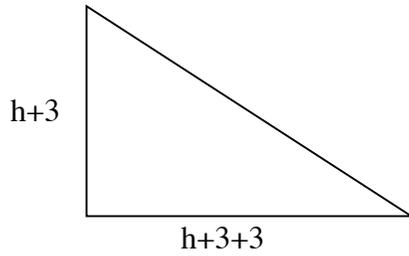
D

$$-8k \ge -16$$

$$k \le 2$$

20.

Let h = height of original triangle
then base of original triange = h+3

New:

Area of new triangle = 14 in^2

$$\frac{1}{2}(h+3)(h+6) = 14$$

$$(h+3)(h+6) = 28$$

$$h^2 + 3h + 6h + 18 = 28$$

$$h^2 + 9h - 10 = 0 \qquad \textbf{A}$$

$$(h+10)(h-1) = 0$$

$$h = -10, \ h = 1$$

Original height = 1 $in.$

Original base = 1 + 3 = 4 $in.$

21.

$$\begin{cases} 2x^2 + y^2 = 1 \\ x - y = 1 \quad \Rightarrow \quad y = x - 1 \end{cases}$$

$$2x^2 + (x-1)^2 = 1$$

$$2x^2 + x^2 - 2x + 1 = 1 \qquad \textbf{B}$$

$$3x^2 - 2x = 0$$

$$x(3x-2) = 0$$

$$x = 0, \ x = \frac{2}{3}$$

6

22.

$$A\,(1,-2),\quad \text{Midpoint } M\,(2,3),\quad B\,(x,y)$$

$$\text{Midpoint} \Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{Midpoint of } \overline{AB} \Rightarrow \left(\frac{1+x}{2}, \frac{-2+y}{2}\right) \Rightarrow (2,3)$$

$$\frac{1+x}{2} = 2, \qquad \frac{-2+y}{2} = 3 \qquad\qquad \mathbf{C}$$

$$1 + x = 4, \qquad -2 + y = 6$$

$$x = 3, \qquad\qquad y = 8$$

$$\text{so } B\,(3,8)$$

23.

$$\text{slope of line} \Rightarrow m = -\frac{1}{3}$$

$$\mathbf{D}$$

$$\text{slope of line perpendicular} \Rightarrow m = 3$$

24.

$$m = \frac{xyk}{z} \qquad\qquad 3 = \frac{(4)(2)k}{6}$$

$$18 = 8k \qquad\qquad \mathbf{B}$$

$$k = \frac{18}{8} = \frac{9}{4}$$

25.

$$2x - 3y = 7 \qquad\qquad\qquad \text{point is } (4,-1)\ ;\ m = \frac{2}{3}$$

$$-3y = -2x + 7 \qquad\qquad\qquad y = mx + b$$

$$y = \frac{2}{3}x - \frac{7}{3} \qquad\qquad\qquad -1 = \left(\frac{2}{3}\right)(4) + b \qquad \mathbf{C}$$

$$\text{slope } m = \frac{2}{3} \qquad\qquad\qquad -1 = \frac{8}{3} + b$$

$$b = -\frac{11}{3}$$

$$\text{slope of parallel line } m = \frac{2}{3} \qquad\qquad \text{so } y = \frac{2}{3}x - \frac{11}{3}$$

26.

Center $\Rightarrow$ $(0,2)$ $\qquad$ $(x-h)^2 + (y-k)^2 = r^2$

radius $= 2$ $\qquad$ $(x-0)^2 + (y-2)^2 = 2^2$

$$x^2 + (y-2)^2 = 4 \qquad \textbf{B}$$

$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + y^2 - 4y = 0$$

27.

$$f(x) = 1 - \sqrt{x}, \qquad g(x) = \frac{1}{x}$$

$$\textbf{D}$$

$$(g \circ f)(x) = g[f(x)] = g\left(1 - \sqrt{x}\right) = \frac{1}{1 - \sqrt{x}}$$

28.

$$f(x) = \frac{x}{x^2 + 1}$$

$$\frac{1}{f(3)} = \frac{1}{\dfrac{3}{(3)^2 + 1}} = \frac{1}{\dfrac{3}{10}} = \frac{10}{3} \qquad \textbf{D}$$

29.

Vertex $\Rightarrow$ $V(0,2)$ $\qquad$ $y = a(x-h)^2 + k$

point on parabola $\Rightarrow$ $(1,0)$ $\qquad$ $y = a(x-0)^2 + 2$

$$y = ax^2 + 2$$

$$0 = a(1)^2 + 2 \qquad \textbf{B}$$

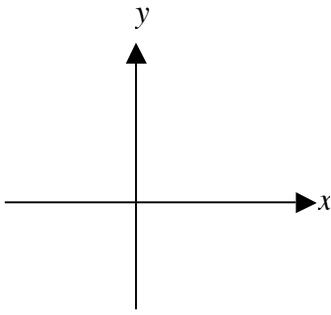$$a = -2$$
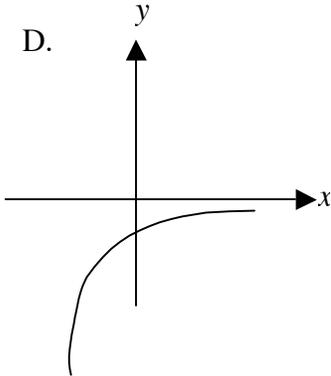
$$y = -2x^2 + 2$$
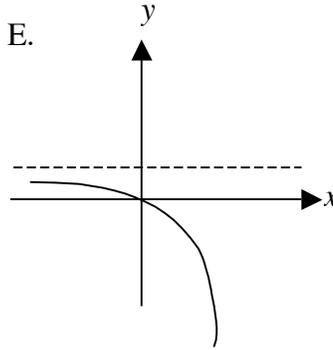
30.

A.

B.

y $\;8$

C.

y

D.

E.

A

31.

$$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b(y^3 y^2) - \log_b y^4$$

B

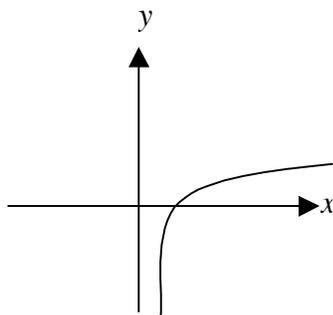$$= \log_b y^5 - \log_b y^4 = \log_b\left(\frac{y^5}{y^4}\right) = \log_b y$$

32.

$f(x) = \log_a x$ if $a > 1$

example : if $a = 2$, then $f(x) = \log_2 x$,

Graph of $y = \log_2 x \implies 2^y = x$

f is increasing, f does not have a as an

x - intercept (the x - int. is $(1,0)$), f does not have

a y - intercept, the domain of f is $(0, \infty)$.

D

33.

9

$$\log\left(\frac{432}{\left(\sqrt{.095}\right)\left(\sqrt[3]{72.1}\right)}\right) = \log\left(\frac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$$

$$= \log 432 - \left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$$

B

$$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}}\right)$$

$$= \log 432 - \frac{1}{2}\log.095 - \frac{1}{3}\log 72.1$$

34.

$$3^{x-5} = 4$$

$$\log 3^{x-5} = \log 4$$

$$(x-5)\log 3 = \log 4 \qquad\qquad \text{C}$$

$$x - 5 = \frac{\log 4}{\log 3}$$

$$x = \frac{\log 4}{\log 3} + 5$$

35.

$$\log_3 \sqrt{2x+3} = 2$$

$$3^2 = \sqrt{2x+3}$$

$$\sqrt{2x+3} = 9$$

$$\left(\sqrt{2x+3}\right)^2 = (9)^2 \qquad\qquad \text{Check}: \quad \sqrt{2(39)+3} = 9 \qquad\qquad \text{C}$$

$$2x + 3 = 81 \qquad\qquad\qquad\qquad\qquad 9 = 9$$

$$2x = 78 \qquad\qquad\qquad\qquad\qquad \text{Check}: \quad \log_3 \sqrt{2(39)+3} = 2$$

$$x = 39 \qquad\qquad\qquad\qquad\qquad\qquad 3^2 = \sqrt{81}$$

36.

10

$\log_3 m = 8$

$$\log_3\left(\frac{\sqrt{mn}}{p^3}\right) = \log_3(mn)^{\frac{1}{2}} - \log_3 p^3$$

$\log_3 n = 10 \quad \Rightarrow$

$$= \log_3\left(m^{\frac{1}{2}}n^{\frac{1}{2}}\right) - \log_3 p^3$$

$\log_3 p = 6$

$$= \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3 \qquad \textbf{A}$$

$$= \frac{1}{2}\log_3 m + \frac{1}{2}\log_3 n - 3\log_3 p$$

$$= \frac{1}{2}(8) + \frac{1}{2}(10) - 3(6)$$

$$= 4 + 5 - 18 = -9$$

37.

$y = 2 + 2^x$

When $x = 0, \quad y = 2 + 2^0$ $\qquad\qquad \textbf{D}$

$\qquad\qquad y = 2 + 1 = 3$

38.

$y = x^2(x - 1)(x + 1)^2$

x - intercepts: $\quad x^2(x-1)(x+1)^2 = 0$

$\qquad\qquad x = 0, \ x = 1, \ x = -1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \textbf{A}$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
x^2	+	+	+	+
$x - 1$	−	−	−	+
$(x + 1)^2$	+	+	+	+
Result	−	−	−	+
	below	below	below	above
	x - axis	x - axis	x - axis	x - axis

39.

11

$$\begin{cases} 2x - 3y = 4 \\ -4x + 6y = 3 \end{cases} \Rightarrow 2E_1 \begin{cases} 4x - 6y = 8 \\ -4x + 6y = 3 \end{cases} \Rightarrow E_1 + E_2$$

Answer is C because:

$$\begin{cases} 4x - 6y = 8 \\ 0 = 11 \end{cases} \Rightarrow \text{No Solution}$$

40.

$$\begin{cases} x + 4y = 3 \\ 2x - 6y = 8 \end{cases} \Rightarrow -2E_1 \begin{cases} -2x - 8y = -6 \\ 2x - 6y = 8 \end{cases} \Rightarrow E_1 + E_2$$

$$\begin{cases} -2x - 8y = -6 \\ -14y = 2 \end{cases} \Rightarrow \quad y = -\frac{2}{14} = -\frac{1}{7} \qquad\qquad \mathbf{E}$$

$$x + 4\left(-\frac{1}{7}\right) = 3$$

$$x = \frac{25}{7} \qquad \text{so } \left(\frac{25}{7}, -\frac{1}{7}\right)$$

41.
let $t =$ number of years after 1980 and let $V =$ value
t is the independent variable and V is the dependent variable

points on line $\Rightarrow$ $(1, 54)$ and $(3, 62)$

$$\text{slope} \Rightarrow m = \frac{62 - 54}{3 - 1} = \frac{8}{2} = 4$$

$$V - V_1 = m(t - t_1) \qquad\qquad \mathbf{A}$$

$$V - 54 = 4(t - 1)$$

$$V - 54 = 4t - 4$$
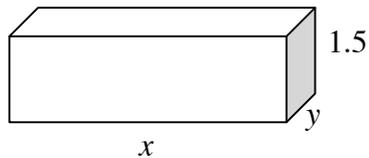
$$V = 4t + 50$$

42.

12

$f(x) = x^2 - 2x + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$$

A

$$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$$

43.

1.5
y
x

$\text{Volume} = 6\,ft.^3$

$xy(1.5) = 6$

$y = \dfrac{6}{1.5x}$ **B**

$y = \dfrac{4}{x}$

44.

$\log_x 2 = 5$

$x^5 = 2$

$\left(x^5\right)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$ **D**

$x = \sqrt[5]{2}$

$x \approx 1.1487$