KEY: DACEB EEABA ED

1. $\dfrac{d}{du}\left(\dfrac{\sqrt[3]{u}}{\sqrt{u}}\right) =$

$= \dfrac{d}{du}\left(\dfrac{u^{1/3}}{u^{1/2}}\right) = \dfrac{d}{du}\left(u^{-1/6}\right)$

$= -\dfrac{1}{6} u^{-7/6}$

A. $3\sqrt{u}$

B. $-\dfrac{1}{3\sqrt[6]{u^7}}$

C. $\dfrac{3}{2\sqrt[6]{u}}$

D. $-\dfrac{1}{6\sqrt[6]{u^7}}$

E. $\dfrac{2}{3\sqrt[6]{u}}$

2. If the line tangent to the curve $y = x^2 + 3x + 1$ at (a, b) passes through the point $(1, 4)$, then the possible values of a are

$y'(a) = 2a + 3$

$y - b = (2a + 3)(x - a)$, $b = a^2 + 3a + 1$

passes through $(1,4)$ implies

$4 - a^2 - 3a - 1 = (2a+3)(1-a) = -2a^2 - a + 3$

Hence, $a^2 - 2a = 0$

and $a = 0$ or 2

A. 0 and 2

B. $\dfrac{1}{4}$ and 2

C. $\dfrac{3}{4}$ and 1

D. 0 and 1

E. $\dfrac{1}{4}$ and $\dfrac{3}{4}$

3. If $h(x) = f(x)\,g(x)$, $f(0) = 1$, $f'(0) = 3$, $g(0) = 1$, and $h'(0) = 5$, then $g'(0) =$

$h'(0) = 5 = f'(0)g(0) + f(0)g'(0)$
$= 3 \times 1 + 1 \times g'(0)$

$\Rightarrow g'(0) = 5 - 3 = 2$

A. 0
B. 1
C. 2
D. -1
E. -2

4. The electric resistance of a wire is $R = \dfrac{\rho\, l}{A}$, where ρ is the specific resistance of the material of the wire, l is the length of the wire and A the area of a cross-section. If specific resistance and cross-sectional area are fixed, the rate of change of R with respect to length is

$R = \left(\frac{\rho}{A}\right) l$

$\frac{dR}{dl} = \frac{\rho}{A}$

A. $\dfrac{l}{A}$
B. $\rho\, l$
C. $-\dfrac{\rho\, l}{A^2}$
D. $-\dfrac{2\rho\, l}{A^2}$
E. $\dfrac{\rho}{A}$

5. $\lim_{x \to 0} \frac{\sin 2x}{\tan 3x} = \lim_{x \to 0} \frac{2x}{3x} \, \frac{\frac{\sin 2x}{2x}}{\frac{\tan 3x}{3x}}$

$$= \frac{\left(\lim_{x\to 0} \frac{2}{3}\right)\left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)}{\frac{\lim_{x\to 0} \frac{\sin 3x}{3x}}{\lim_{x\to 0} \cos 3x}} = \frac{\frac{2}{3} \times 1}{\frac{1}{1}} = \frac{2}{3}$$

A. $= 1$

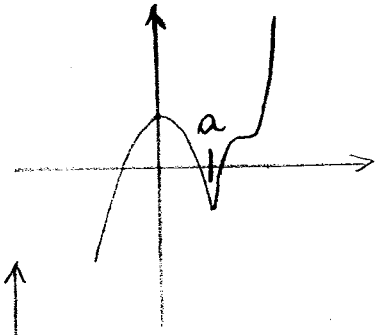
B. $= \frac{2}{3}$

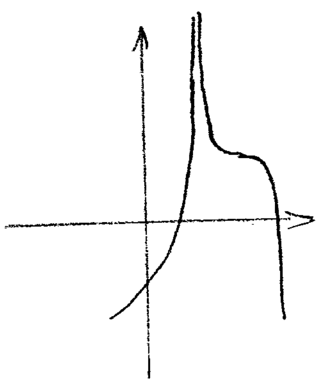
C. $= \frac{1}{3}$

D. $= \frac{1}{6}$

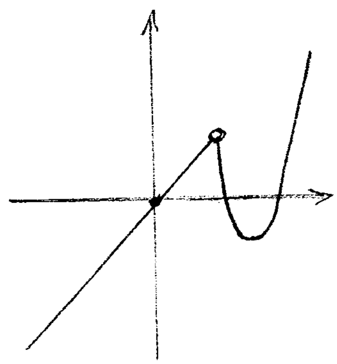
E. does not exist

6. If the graph of f is sketched on the right, which is the graph of f'?

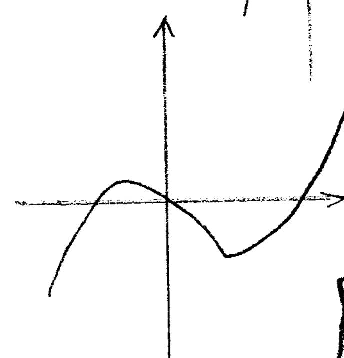




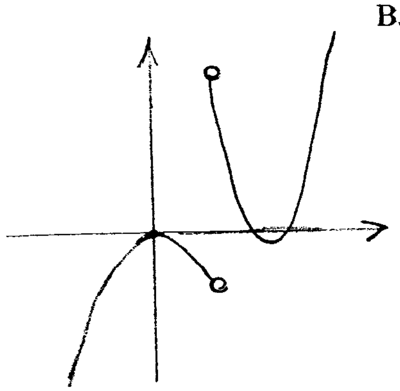
A.



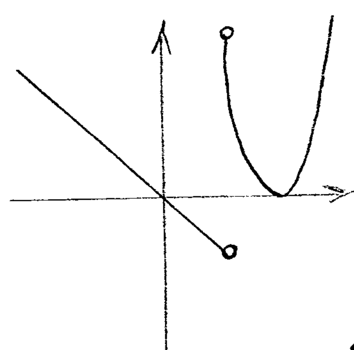
B.



C.



D.



E.

$f'(x) > 0$ for $x < 0$ AND $x > a$

ONLY (E)

$f'(x) < 0$ for $0 < x < a$

f' discontinuous at $x = a$

7. If $y = \sqrt[3]{\dfrac{t^3+1}{t^3-1}}$, then $\dfrac{dy}{dt} =$

$$\frac{d}{dt}\left(\frac{t^3+1}{t^3-1}\right)^{1/3} = \frac{1}{3}\left(\frac{t^3-1}{t^3+1}\right)^{2/3} \frac{3t^2(t^3-1)-3t^2(t^3+1)}{(t^3-1)^2}$$

$$= \frac{-2t^2}{\sqrt[3]{(t^3-1)^4\,(t^3+1)^2}}$$

A. $\dfrac{2t^5}{\sqrt[3]{(t^3-1)^4(t^3+1)^2}}$

B. $-\dfrac{2t^2}{\sqrt[3]{(t^3-1)^2(t^3+1)^4}}$

C. $\dfrac{6t^2}{\sqrt[3]{(t^3-1)^4(t^3+1)^2}}$

D. $-\dfrac{6t^2\,(t^3+1)}{\sqrt[3]{(t^3-1)^2}}$

E. $-\dfrac{2t^2}{\sqrt[3]{(t^3-1)^4(t^3+1)^2}}$

8. An equation of the line tangent to the graph of $x\ \cos y + y\ \cos x = 1$ at the point $(0,1)$ is

$$\cos y - x\sin y\, y' + y'\cos x - y\sin x = 0$$

$$y' = \frac{y\sin x - \cos y}{\cos x - x\sin y}$$

at $(0,1)$ $\quad y' = \dfrac{-\cos 1}{\cos 0} = -\cos 1$

$$y - 1 = -\cos 1\,(x - 0)$$

A. $(\cos 1)\,x + y = 1$

B. $x + y = 1$

C. $-(\sin 1)\,x + y = 1$

D. $x - y = 1$

E. $(\tan 1)\,x + y = 1$

9. Evaluate $\lim_{s\to\frac{\pi}{3}} \frac{\tan s - \sqrt{3}}{s - \frac{\pi}{3}}$. $= \frac{d}{ds} \tan s \Big|_{s=\frac{\pi}{3}}$

$= \sec^2 \frac{\pi}{3} = \frac{1}{\cos^2 \frac{\pi}{3}} = \frac{1}{(1/2)^2} = 4$

A. 2
B. 4
C. $\frac{1}{2}$
D. $\frac{1}{4}$
E. $\sqrt{3}$

10. If $f(x) = \cosh 2x$, then $f''(x) =$

$f'(x) = (\sinh 2x)(2)$

$f''(x) = 2(\cosh 2x)(2)$

$= 4 \cosh 2x$

A. $4\cosh 2x$
B. $-2\cosh 2x$
C. $-4\sinh 2x$
D. $2\sinh 2x$
E. $2\sinh 4x$

11. If $f(x) = \sqrt{x}\, e^{x^2}\, (x^2 - 7)^9$, then $f'(x) =$

$$\ln f(x) = \frac{1}{2}\ln x + x^2 + 9\ln(x^2 - 7)$$

$$\frac{d}{dx}\ln f(x) = \frac{1}{2x} + 2x + \frac{9 \times 2x}{x^2 - 7}$$

A. $\sqrt{x}\, e^{x^2}\, (x^2-7)^9 \left[-\frac{1}{2x} + 2x + \frac{2x}{9(x^2-7)}\right]$

B. $\sqrt{x}\, e^{x^2}\, (x^2-7)^9 \left[\frac{1}{2x} + 2x + \frac{2x}{9(x^2-7)^8}\right]$

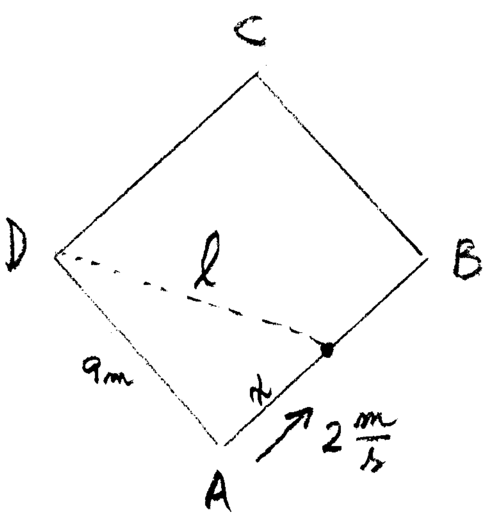
C. $\sqrt{x}\, e^{x^2}\, (x^2-7)^9 \left[\frac{1}{2x} + 1 + \frac{18x}{x^2-7}\right]$

D. $\sqrt{x}\, e^{x^2}\, (x^2-7)^9 \left[-\frac{1}{2x} + 2x + \frac{18x}{(x^2-7)^8}\right]$

E. $\sqrt{x}\, e^{x^2}\, (x^2-7)^9 \left[\frac{1}{2x} + 2x + \frac{18x}{x^2-7}\right]$

12. A mini-baseball diamond is a square ABCD with side 9 meters. A batter hits the ball at A and runs toward first base B with a speed of 2 m/s. At what rate is his distance from third base D increasing when he is two-thirds of the way to first base?

$\frac{dx}{dt} = 2$, $\quad \ell^2 = 9^2 + x^2$



$$\Downarrow$$

$$2\ell\frac{d\ell}{dt} = \frac{d}{dt}(9^2) + 2x\frac{dx}{dt}$$

$$\Downarrow$$

$$\frac{d\ell}{dt} = \frac{x}{\ell}\frac{dx}{dt} = \frac{2x}{\ell}$$

For $x = 6$, $\ell = \sqrt{9^2 + 6^2} = 3\sqrt{13}$

and $\frac{d\ell}{dt} = \frac{2 \times 6}{3\sqrt{13}} = \frac{4}{\sqrt{13}}$

A. $\frac{3}{\sqrt{13}}$ m/s

B. 2 m/s

C. 4 m/s

D. $\frac{4}{\sqrt{13}}$ m/s

E. $\frac{24}{\sqrt{13}}$ m/s