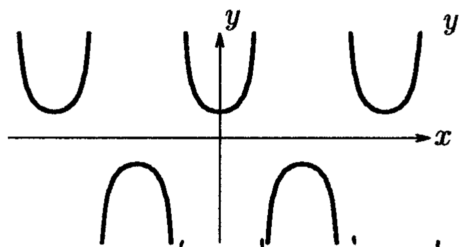
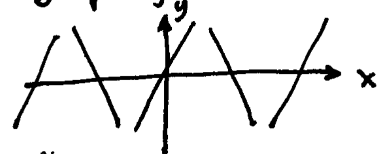
1. Given the graph of $y = f'(x)$, choose which graph could represent the graph of $y = f(x)$.



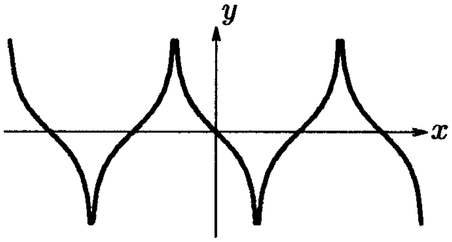
$y = f'(x)$

$\Rightarrow$ slope of f is: pos. | neg. | pos. | neg. | pos. $\Rightarrow$

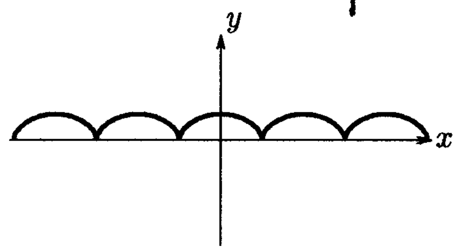
graph of f is like:



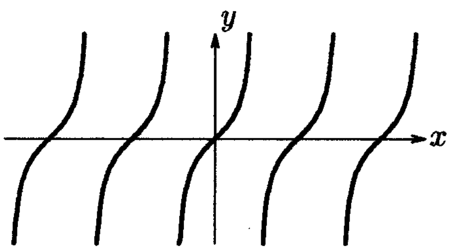
A.



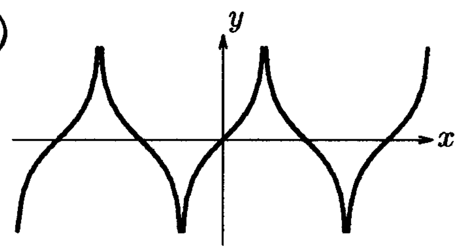
B.



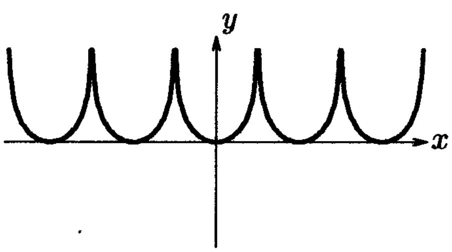
C.



D.



E.



2. At how many different values of x does the curve $y = x^5 + 2x$ have a tangent line parallel to the line $y = x$.

A. 0
B. 1
C. 2
D. 3
E. 4

derivative of $y = x^5 + 2x$ must equal 1.

$\Rightarrow 5x^4 + 2 = 1$

$\Rightarrow 5x^4 = -1$

$\Rightarrow$ no solution.

$\Rightarrow$ no values of x satisfy problem

3. Given the table

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-1	2	-2
1	1	2	3	4
2	-4	-3	-2	-1
3	2	4	3	0

$x=1 \longrightarrow$

find $\frac{d}{dx}\left(\frac{f(x)}{f(x)+g(x)}\right)$ when $x = 1$.

A. 0
B. $\frac{1}{8}$
C. $\frac{1}{2}$
D. $\frac{14}{16}$
E. -2

$$\frac{d}{dx}\left(\frac{f(x)}{f(x)+g(x)}\right) = \frac{(f'(x))(f(x)+g(x)) - (f(x))(f'(x)+g'(x))}{(f(x)+g(x))^2}$$

$$\Rightarrow \left.\frac{d}{dx}\left(\frac{f(x)}{f(x)+g(x)}\right)\right|_{x=1}$$

$$= \frac{(2)(1+3) - (1)(2+4)}{(1+3)^2} = \frac{8-6}{16} = \frac{1}{8}$$

4. If $f(x) = \tan^{-1} x$ then $f'(2) =$

$$f'(x) = \frac{1}{1+x^2} \longrightarrow f'(2) = \frac{1}{1+4}$$

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{1}{\sqrt{3}}$

E. $f'(2)$ does not exist.

5. If $f(x) = \pi^x$ then $f'(x) =$

$$f'(x) = \pi^x \cdot \ln \pi$$

Note: $f(x) = \pi^x = (e^{\ln \pi})^x = e^{(\ln \pi)(x)}$

$$\rightarrow f'(x) = e^{(\ln \,)(x)} \cdot \ln \pi = \pi^x \cdot \ln \pi$$

A. π^x

B. $\pi^x \ln x$

C. $e^{x \ln \pi}$

D. $\pi^x \ln \pi$

E. $\pi^{\ln x}$

6. If $F(x) = g(x^2)$ then $F''(x) =$

$$F'(x) = g'(x^2) \cdot 2x$$

$$\rightarrow F''(x) = (g''(x^2) \cdot 2x)(2x) + (g'(x^2))(2)$$

A. $2x^2 g''(x) + 2(g(x))^2$

B. $g''(2x)$

C. $2xg''(x^2)$

D. $4xg''(x) + g'(x^2)$

E. $4x^2 g''(x^2) + 2g'(x^2)$

7. If $f(x) = e^x \tan x$ then $f'\left(\frac{\pi}{4}\right) =$

A. $-2e^{\pi/4}$
B. $3e^{\pi/4}$
C. $e^{\pi/4}$
D. $2e^{\pi/4}$
E. $-e^{\pi/4}$

$$f'(x) = (e^x)(\tan x) + (e^x)(\sec^2 x)$$

$$f'\left(\tfrac{\pi}{4}\right) = (e^{\pi/4})(1) + (e^{\pi/4})(2)$$

$$= 3e^{\pi/4}$$

8. If $f(x) = \ln(e^{x^2} + 1)$ then $f'(1) =$

A. $\dfrac{2e+1}{e+1}$
B. $\dfrac{1}{e+1}$
C. $\dfrac{e}{e+1}$
D. $\dfrac{2e}{e+1}$
E. $\dfrac{1}{(e+1)^2}$

$$f'(x) = \left(\frac{1}{e^{x^2}+1}\right)\left(e^{x^2}\right)(2x)$$

$$f'(1) = \left(\frac{1}{e+1}\right)(e)(2)$$

$$= \frac{2e}{e+1}$$

9. If $x \sin y = y \cancel{\sin} \cos x$ then the value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{4}$, $y = \dfrac{\pi}{4}$ is

A. $\dfrac{1-\pi/4}{1+\pi/4}$
B. -1
C. 0
D. 1
E. $\dfrac{1+\pi/4}{1-\pi/4}$

differentiating w.r.t. $x \Longrightarrow$

$$(1)(\sin y) + (x)(\cos y)(dy/dx) = (dy/dx)(\cos x) + (y)(-\sin x)$$

Substituting $x = \pi/4$ and $y = \pi/4 \Longrightarrow$

$$\frac{1}{\sqrt{2}} + \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}\cdot dy/dx = dy/dx\cdot\frac{1}{\sqrt{2}} - \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}$$

Multiply by $\sqrt{2} \longrightarrow$

$$1 + \frac{\pi}{4}\, dy/dx = dy/dx - \frac{\pi}{4}$$

$$\longrightarrow \frac{\pi}{4}\, dy/dx - dy/dx = -\frac{\pi}{4} - 1 \longrightarrow \frac{dy}{dx} = \frac{-\left(\frac{\pi}{4}+1\right)}{\frac{\pi}{4}-1} = \frac{\frac{\pi}{4}+1}{-\frac{\pi}{4}+1}$$

10. A radio-active isotope has a half-life of 103 years. How long in years will it take for $\frac{9}{10}$ of a sample of 7 grams to decay?

A. $103(\ln 10 - \ln 9)$

B. $\ln 103 - \ln 7$

C. $\dfrac{\ln 103}{\ln(9/10)}$

D. $7(\ln 10 - \ln 2)$

E. $\dfrac{103 \ln 10}{\ln 2}$

First find k: $A(t) = A(0)e^{kt}$,

$A(103) = \frac{1}{2}A(0) = A(0)e^{103k} \rightarrow \frac{1}{2} = e^{103k}$

$\rightarrow \ln\frac{1}{2} = 103k \rightarrow k = \frac{1}{103}\ln\frac{1}{2}$

Now find t where $A(t) = \frac{1}{10}A(0)$.

$\rightarrow \frac{1}{10}\cancel{A(0)} = \cancel{A(0)}e^{\frac{1}{103}\cdot\ln\frac{1}{2}\cdot t}$

$\rightarrow \ln\frac{1}{10} = \frac{1}{103}\cdot\ln\frac{1}{2}\cdot t \rightarrow t = 103\frac{\ln\frac{1}{10}}{\ln\frac{1}{2}} = 103\left(\frac{-\ln 10}{-\ln 2}\right) = 103\frac{\ln 10}{\ln 2}$.

11. The height of a particle is given by

$$h(t) = 4t^3 - 9t^2 + 6t + 2, \qquad t \geq 0.$$

When is the particle moving in the downward direction?

A. $\frac{1}{2} < t < 1$

B. $1 < t$

C. $0 < t < \frac{1}{2}$

D. $0 < t < \frac{1}{2}$ and $1 < t$

E. It never moves in the downward direction

$h'(t) = 12t^2 - 18t + 6 = 6(2t^2 - 3t + 1)$

$= 6(2t-1)(t-1)$

$h'(t) = 0 \rightarrow t = \frac{1}{2}, 1$.

t	0 to 1/2	1/2 to 1	1 to ∞
$2t-1$	−	+	+
$t-1$	−	−	+
$h'(t)$	+	−	+

$\rightarrow h'(t) < 0$ when $\frac{1}{2} < t < 1$.

12. If $f(x) = e^{-3x} + x^{17} + 1$ then $f^{(20)}(0) =$

A. $3^{20} + 17!$

B. -3^{20}

C. 3^{20}

D. 1

E. -1

The 20-th derivative of $x^{17} + 1$ is 0.

$f'(x) = -3e^{-3x} + \cdots$

$f''(x) = (-1)^2(3)^2 e^{-2x} + \cdots$

$f'''(x) = (-1)^3(3)^3 e^{-2x} + \cdots$

$\rightarrow f^{(20)}(0) = (-1)^{20}(3)^{20}e^{0} = 3^{20}$

13. The width of a rectangle is increasing at 3 feet per second and its length is decreasing at 2 feet per second. When the width is 12 and its length is 8 what is the rate of change of the area of the rectangle?

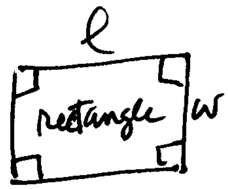
A. 20

B. 52

C. -16

D. -84

E. 0



$$\text{Area} = A = \ell w.$$

$$\rightarrow \frac{dA}{dt} = \ell \cdot \frac{dw}{dt} + \frac{d\ell}{dt} \cdot w$$

$$= (8)(3) + (-2)(12)$$

$$= 24 - 24$$

$$= 0$$

14. Sue leaves the bell-tower at 9:00 am heading north at 2 meters per second. John leaves the bell-tower one second later heading east also at 2 meters per second. How fast in meters per second is the distance between Sue and John increasing 3 seconds after John left?

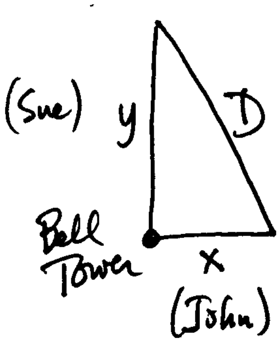
A. 4

B. 3.6

C. 3.2

D. 2.8

E. 2.4



$$D^2 = x^2 + y^2 \text{ and } \frac{dx}{dt} = \frac{dy}{dt} = 2.$$

$$\rightarrow 2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\rightarrow \frac{dD}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{D}$$

At the time in question, $y = 8$ and $x = 6 \rightarrow D = \sqrt{8^2 + 6^2} = 10$.

$$\rightarrow \frac{dD}{dt} = \frac{(6)(2) + (8)(2)}{10} = \frac{12+16}{10} = \frac{28}{10} = 2.8$$