1. Let $y = x^4 + x^2 + 1$, $x = 1$, and $dx = 2$. Then $dy =$

A. 4
B. 8
C. 12
D. 16
E. 20

$dy = \frac{dy}{dx} \cdot dx$

$= (4x^3 + 2x)\,dx$

$x=1, dx=2 \Rightarrow dy = (4 \cdot 1^3 + 2 \cdot 1)(2) = 12$

2. The function $f(x) = x^3 + ax^2 + bx$ has critical numbers at $x = 1$ and $x = 3$. Find a and b

A. $a = -9,\ b = 6$
B. $a = -8,\ b = 7$
C. $a = -7,\ b = 8$
D. $a = -6,\ b = 9$
E. $a = -6,\ b = 6$

$f'(x) = 3x^2 + 2ax + b = 0.$

$x=1 \rightarrow 3 + 2a + b = 0 \quad (1)$

$x=3 \rightarrow 27 + 6a + b = 0 \quad (2)$

$(2)-(1) \rightarrow 24 + 4a = 0 \rightarrow a = -6$

$a=-6, (1) \rightarrow 3 - 12 + b = 0 \rightarrow b = 9$

3. The difference between the maximum and minimum values of the function $f(x) = x^3 - x^2 - x$ on the interval $[-1, 2]$ is

A. $\frac{5}{27}$
B. 1
C. $\frac{22}{27}$
D. 2
E. 3

$f'(x) = 3x^2 - 2x - 1$

$f'(x) = 0 \rightarrow (3x+1)(x-1) = 0 \rightarrow x = -\frac{1}{3}, 1.$

$f(-1) = -1$ min

$f(-\frac{1}{3}) = \frac{5}{27}$

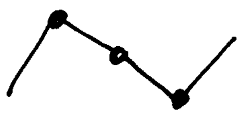
$f(1) = -1$ min

$f(2) = 2$ max

max − min $= 2 - (-1) = 3$

4. $f'(x) = (x-2)x^2(x+3)$. Which line of the following table describes the behavior of f at $x = -3, 0, 2$.

	$-\infty$ to -3	-3 to 0	0 to 2	2 to ∞
$x+3$	$-$	$+$	$+$	$+$
x^2	$+$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
$f'(x)$	$+$	$-$	$-$	$+$

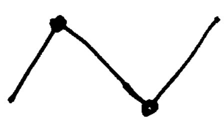
f


	-3	0	2
A	local max	neither local max nor local min	local min
B	local max	local min	local max
C	local min	local max	local min
D	local min	neither local max nor local min	local min
E	neither local max nor local min	local max	local min

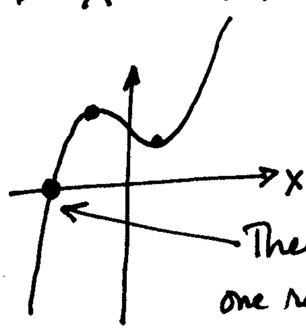
5. $f(x) = x^7 - 7x + 17$ has exactly two critical points which are at $(-1, 23)$ and $(1, 11)$. The number of real zeros of f is

$f'(x) = 7x^6 - 7 = 0 \rightarrow x = \pm 1$.

	$-\infty$ to -1	-1 to 1	1 to ∞
$7(x^6-1)$	$+$	$-$	$+$

f


$f(-1) = 23$, $f(1) = 11$



There is only one real zero of f

A. 0
B. 1
C. 2
D. 6
E. 7

6. The largest interval on which the function $f(x) = x\sin x + \cos x$, $0 \le x \le \pi$ is increasing is

$f'(x) = \sin x + x\cos x - \sin x = x\cos x$

$f'(x) = 0 \longrightarrow x = 0, \pi/2$

	0 to $\pi/2$	$\pi/2$ to π
x	$+$	$+$
$\cos x$	$+$	$-$
$f'(x)$	$+$	$-$

$f'(x) > 0$

A. $[\frac{\pi}{3}, \frac{\pi}{2}]$
B. $[0, \frac{\pi}{3}]$
C. $[\frac{\pi}{3}, \pi]$
D. $[0, \frac{\pi}{2}]$
E. $[\frac{\pi}{2}, \pi]$

7. What is the length of the largest interval on which the function $f(x) = \dfrac{x}{x^2+1}$ is increasing?

$$f'(x) = \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2} = 0 \rightarrow x = \pm 1$$

	$-\infty$ to -1	-1 to 1	1 to ∞
$1-x^2$	$-$	$+$	$-$
$f'(x)$	$-$	$+$	$-$

length of interval $= 2$

A. 1
B. 4
C. $\sqrt{3}$
D. 2
E. ∞

8. On what interval is the graph of $f(x) = (1 - \frac{1}{x})^2$ concave downward?

$$f'(x) = 2\left(1-\frac{1}{x}\right)\left(\frac{1}{x^2}\right) = 2\left(\frac{x-1}{x^3}\right)$$

$$f''(x) = 2\left(\frac{(1)(x^3) - (x-1)(3x^2)}{x^6}\right) = 2\left(\frac{-2x+3}{x^6}\right)$$

	$-\infty$ to 0	0 to $3/2$	$3/2$ to ∞
$-2x+3$	$+$	$+$	$-$
x^6	$+$	$+$	$+$
$f''(x)$	$+$	$+$	$-$
f			

A. $(-\infty, 0)$
B. $(\frac{3}{2}, \infty)$
C. $(0, \infty)$
D. $(-\infty, -1)$
E. $(1, \frac{3}{2})$

9. $\lim\limits_{x \to \frac{\pi}{2}} (\pi^2 - 4x^2)\tan x =$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\pi^2 - 4x^2}{\cot x} = \frac{0}{0}$$

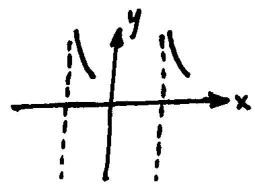
$$= \lim_{x \to \frac{\pi}{2}} \frac{-8x}{-\csc^2 x} = \frac{-4\pi}{-1} = 4\pi$$

A. 0
B. 2π
C. 4π
D. ∞
E. does not exist

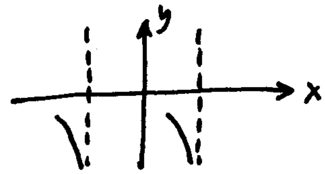
10. Given the following information, select a graph that could be the graph of $y = f(x)$

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to-\infty} f(x) = 0$ $\longrightarrow$ $y=0$ is the horizontal asymptote

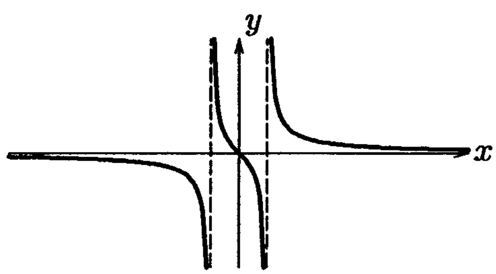
$\lim\limits_{x\to-1^+} f(x) = \lim\limits_{x\to1^+} f(x) = \infty$ $\longrightarrow$



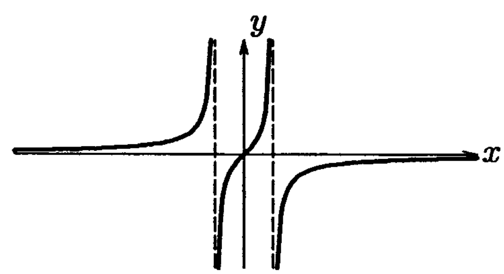
$\lim\limits_{x\to-1^-} f(x) = \lim\limits_{x\to1^-} f(x) = -\infty$ $\longrightarrow$



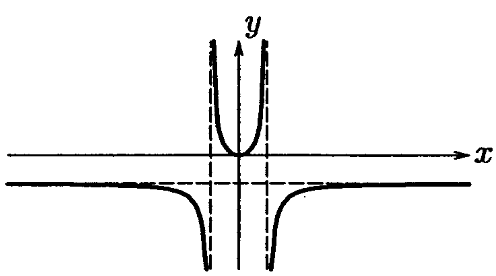
A.



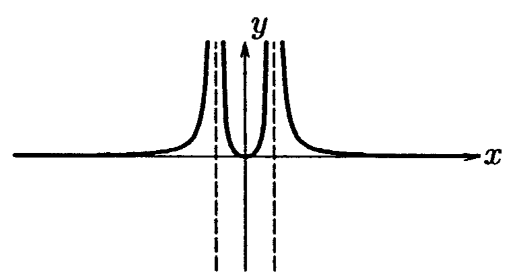
B.



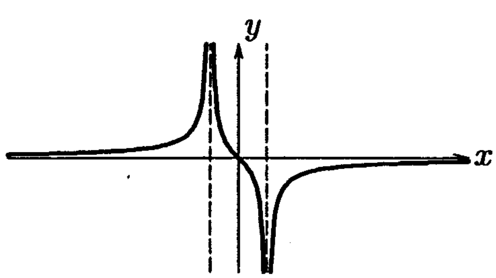
C.



D.



E.



11. Positive real numbers x and y are to be chosen so that $x + y = 60$ and xy^2 is as large as possible. What is the value of x?

$P = xy = (x)(60-x^2) = 3600x - 120x^2 + x^3$

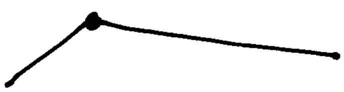
domain is $0 < x < 60$.

$P'(x) = 3600 - 240x + 3x^2$
$= (60-x)(60-3x) = 0 \rightarrow x = 20, 60$

A. 16
B. 18
C. 20
D. 24
E. 60

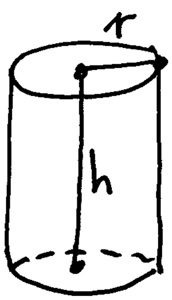
x:	0 to 20	20 to 60
$60-x$	+	+
$60-3x$	+	−
$P'(x)$	+	−

P



max P when $x = 20$

12. A closed container is to be constructed in the shape of a right circular cylinder. If the volume is to be 24π cm^3 find the ratio of the height to the radius which will minimize the surface area.



$V = \pi r^2 h = 24\pi \rightarrow h = \frac{24}{r^2}$

Area $= A = 2\pi r^2 + 2\pi r h$
$= 2\pi r^2 + 2\pi r\left(\frac{24}{r^2}\right)$

A. 2 to 1
B. 3 to 1
C. 3 to 2
D. 5 to 3
E. 2 to 3

$A(r) = 2\pi r^2 + 48\pi \cdot \frac{1}{r}$, $r > 0$.

$A'(r) = 4\pi r - 48\pi \cdot \frac{1}{r^2} = \frac{4\pi r^3 - 48\pi}{r^2} = 0 \rightarrow r = \sqrt[3]{12}$

r:	0 to $\sqrt[3]{12}$	$\sqrt[3]{12}$ to ∞
	−	+

$4\pi r^3 - 48\pi = 4\pi(r^3 - 12)$

A

minimum A at $r = \sqrt[3]{12}$, h to r $= 2\sqrt[3]{12}$ to $\sqrt[3]{12}$
$= 2$ to 1.

$h(\sqrt[3]{12}) = \frac{24}{12^{2/3}} = \frac{2 \cdot 12}{12^{2/3}} = 2 \cdot 12^{1/3}$.

13. If $f''(x) = 6x^2 - 1$, $f(0) = 1$, $f'(0) = \frac{3}{2}$ then $f(1) =$

A. $\frac{1}{2}$
B. 1
C. $\frac{3}{2}$
D. 2
E. $\frac{5}{2}$

$f''(x) = 6x^2 - 1$

$\rightarrow f'(x) = 2x^3 - x + c_1$

$f'(0) = 0 - 0 + c_1 = \frac{3}{2} \rightarrow c_1 = \frac{3}{2}$

$f'(x) = 2x^3 - x + 3/2$

$\rightarrow f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 + \frac{3}{2}x + c_2$

$f(0) = 0 - 0 + 0 + c_2 = 1 \rightarrow c_2 = 1$

Therefore, $f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 + \frac{3}{2}x + 1$

and $f(1) = \frac{1}{2} - \frac{1}{2} + \frac{3}{2} + 1 = \frac{5}{2}$

14. If $f''(x) = x^{-2}$ and $f(1) = 0$, $f(2) = 0$, then $f\left(\frac{1}{2}\right) =$

A. $\frac{9}{2}\ln 2$
B. $\frac{7}{2}\ln 2$
C. $\frac{5}{2}\ln 2$
D. $\frac{3}{2}\ln 2$
E. $\frac{1}{2}\ln 2$

$f''(x) = x^{-2} \rightarrow f'(x) = -\frac{1}{x} + c_1$

$\rightarrow f(x) = -\ln|x| + c_1 x + c_2$.

$f(1) = 0 + c_1 + c_2 = 0$ (1)

$f(2) = -\ln 2 + 2c_1 + c_2 = 0$ (2)

(1) $\rightarrow c_2 = -c_1$

substitute (1) into (2) $\rightarrow -\ln 2 + 2c_1 - c_1 = 0$

$\rightarrow -\ln 2 + c_1 = 0 \rightarrow c_1 = \ln 2 \rightarrow c_2 = -\ln 2$

Therefore $f(x) = -\ln|x| + (\ln 2)x - \ln 2$

and $f\left(\frac{1}{2}\right) = -\ln\frac{1}{2} + \frac{1}{2}\ln 2 - \ln 2$

$= \ln 2 + \frac{1}{2}\ln 2 - \ln 2 = \frac{1}{2}\ln 2$