1. Solve the inequality $x^3 \geq 2x^2 + 3x$

A. $x \geq 3$

B. $-1 \leq x \leq 0$ or $x \geq 3$

C. $x \leq -1$ or $0 \leq x \leq 3$

D. $-3 \leq x \leq 0$ or $x \geq 1$

E. $x \leq -3$ or $0 \leq x \leq 1$

$\rightarrow x^3 - 2x^2 - 3x \geq 0$

$\rightarrow x(x-3)(x+1) \geq 0$

	$-\infty$ to -1	-1 to 0	0 to 3	3 to ∞
x	$-$	$-$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$	$+$
$(x)(x-3)(x+1)$	$-$	$+$	$-$	$+$
		≥ 0		≥ 0

2. $\sin(\cos^{-1} x) =$

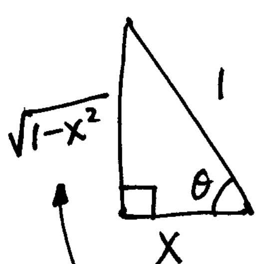
A. $\tan x$

B. $\dfrac{x}{\sqrt{1-x^2}}$

C. $\dfrac{\sqrt{1-x^2}}{x}$

D. $\dfrac{1}{\sqrt{1-x^2}}$

E. $\sqrt{1-x^2}$



let $\cos^{-1} x = \theta$

then $\cos\theta = x$

this side from pythagorean theorem.

$\therefore \sin(\cos^{-1} x) = \sin\theta = \sqrt{1-x^2}$

3. If $f(x) = \frac{1}{2-x}$ and $g(x) = \frac{x}{x-3}$ then the domain of $f \circ g$ is

A. $x = \frac{5}{3}, 2, 3$

B. $x \neq \frac{5}{3}, 2$

C. $x \neq 6$

D. $x \neq 3, 6$

E. $x \neq 2, 3, 6$

domain of g is $x \neq 3$.

domain of f is $x \neq 2$.

$\therefore g(x) \neq 2 \rightarrow \frac{x}{x-3} \neq 2$

$\rightarrow x \neq 2x - 6$

$\rightarrow -x \neq -6$

$\rightarrow x \neq 6$

$\therefore$ domain of $f \circ g$ is $x \neq 3, 6$

4. Express the quantity $x + \frac{\ln x}{2} - 2\ln(x+1)$ as a single logarithm.

A. $\ln \frac{x\sqrt{x}}{(x+1)^2}$

B. $\ln(\sqrt{x}(x+1)^2)$

C. $\ln(x+\sqrt{x}-(x+1)^2)$

D. $\ln \frac{x}{4(x+1)}$

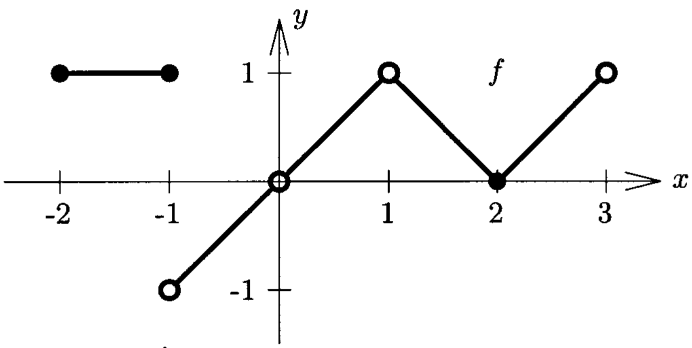
E. $\ln \frac{e^x\sqrt{x}}{(x+1)^2}$

$x + \frac{\ln x}{2} - 2\ln(x+1)$

$= \ln(e^x) + \ln(x^{\frac{1}{2}}) - \ln((x+1)^2)$

$= \ln\left(\frac{e^x x^{1/2}}{(x+1)^2}\right)$

5. Given the following graph of the function f, which statement is true?



A. $\lim_{x \to 0} f(x)$ does not exist

B. $f(-1)$ does not exist

C. $\lim_{x \to 1} f(x) = f(3)$

D. $\lim_{x \to 1} f(x) = 1$

E. $\lim_{x \to -1} f(x) = -1$

A. $\lim_{x \to 0} f(x) = 0 \Rightarrow$ F

B. $f(-1) = 1 \Rightarrow$ F

C. $\lim_{x \to 1} f(x) = 1$, but $f(3)$ does not exist $\Rightarrow$ F

D. $\lim_{x \to 1} f(x) = 1 \Rightarrow \boxed{T}$

E. $\lim_{x \to -1} f(x)$ does not exist $\Rightarrow$ F

6. Find the number a such that $\lim_{x \to 1} \dfrac{x^2 + 3ax + a + 3}{x^2 + 2x - 3}$ exists and is finite.

A. $a = -2$

B. $a = -1$

C. $a = 0$

D. $a = 1$

E. $a = 2$

Since $\lim_{x \to 1} x^2 + 2x - 3 = 0$, Then limit will exist if $\lim_{x \to 1} x^2 + 3ax + a + 3 = 0$.

Now $\lim_{x \to 1} x^2 + 3ax + a + 3 = 4a + 4$.

$4a + 4 = 0 \rightarrow a = -1$.

Note: if $a = -1$ then $\lim_{x \to 1} \dfrac{x^2 + 3ax + a + 3}{x^2 + 2x - 3}$

$$= \lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 + 2x - 3} = \lim_{x \to 1} \frac{x-2}{x+3} = -\frac{1}{4}.$$

7. The tangent line to $y = f(x)$ at $(3, -5)$ passes through the point $(-2, 4)$. Find $f'(3)/f(3)$.

tangent line passes thru $(3,-5)$ and $(-2,4)$

$\Rightarrow f'(3) = \dfrac{-5-4}{3-(-2)} = \dfrac{-9}{5}$

$\therefore \dfrac{f'(3)}{f(3)} = \dfrac{-\frac{9}{5}}{-5} = \left(-\dfrac{9}{5}\right)\left(\dfrac{1}{-5}\right) = \dfrac{9}{25}$

A. $\frac{9}{10}$

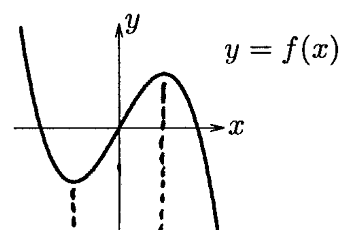
B. $\frac{9}{15}$

C. $-\frac{9}{15}$

D. $\frac{9}{25}$

E. $-\frac{18}{25}$

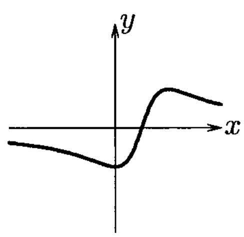
8. Given the graph of the function f



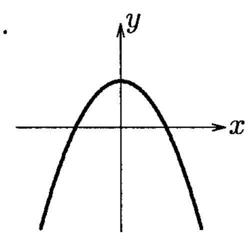
$f'(x) \quad - \; 0 \; + \; 0 \; -$ $\Longrightarrow$ graph of f' is B.

the graph of its derivative looks most like

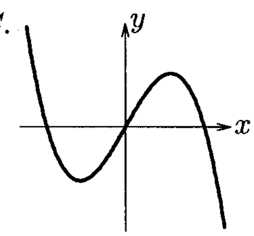
A.



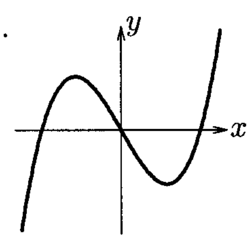
B.



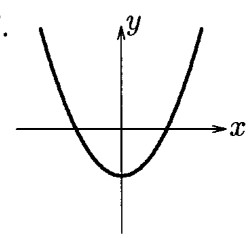
C.



D.



E.



9. $\lim_{h\to 0} \dfrac{e^h - 1}{h} = \dfrac{0}{0}$

$$\stackrel{\frac{0}{0}}{=} \lim_{h\to 0} \frac{e^h}{1} = \frac{1}{1} = 1$$

A. 1

B. 0

C. e

D. -1

E. none of the above

10. If $g(x) = \dfrac{e^x}{x+e^x}$ then $g'(x) =$

$$g'(x) = \frac{(e^x)(x+e^x) - (e^x)(1+e^x)}{(x+e^x)^2}$$

$$= \frac{xe^x + e^{2x} - e^x - e^{2x}}{(x+e^x)^2}$$

$$= \frac{xe^x - e^x}{(x+e^x)^2}$$

$$= \frac{e^x(x-1)}{(x+e^x)^2}$$

A. $\dfrac{e^{2x}(x-1)}{(x+e^x)^2}$

B. $\dfrac{e^x(x-1)}{(x+e^x)^2}$

C. $\dfrac{e^x}{1+e^x}$

D. $\dfrac{2e^{2x} + (x-1)e^x}{(x+e^x)^2}$

E. $\dfrac{1}{(x+1)^2}$

11. The frequency of vibrations of a vibrating violin string is $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}}$, where L is the length, T the tension, and ρ the linear density. The rate of change of the frequency with respect to the tension is

$$f = \frac{1}{2L\sqrt{\rho}} \cdot \sqrt{T}$$

$$\frac{df}{dt} = \frac{1}{2L\sqrt{\rho}} \cdot \frac{1}{2\sqrt{T}} = \frac{1}{4L\sqrt{\rho T}}$$

A. $\dfrac{1}{4L\sqrt{\rho T}}$

B. $\dfrac{1}{4L\sqrt{\dfrac{T}{\rho}}}$

C. $-\dfrac{1}{2L^2}\sqrt{\dfrac{T}{\rho}}$

D. $-\dfrac{1}{4\rho L}\sqrt{\dfrac{T}{\rho}}$

E. $\dfrac{1}{2L\sqrt{\rho}}$

12. The slope of the tangent to the curve $y = (2 + \sin x)\cos x$ at $\left(\dfrac{5\pi}{4}, \dfrac{1}{2} - \sqrt{2}\right)$ is

$$\frac{dy}{dx} = (\cos x)(\cos x) + (2 + \sin x)(-\sin x)$$

$$= \cos^2 x - 2\sin x - \sin^2 x$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{4}} = \left(-\frac{1}{\sqrt{2}}\right)^2 - 2\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \sqrt{2} - \frac{1}{2} = \sqrt{2}$$

A. 0

B. $\dfrac{1}{2}$

C. $\dfrac{1}{\sqrt{2}}$

D. $\sqrt{2}$

E. -1

13. $\frac{d}{dx} e^{\sqrt{x}} =$

A. $e^{\sqrt{x}}$

B. $\frac{e^{\sqrt{x}}}{2}$

C. $e^{\frac{1}{2\sqrt{x}}}$

D. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

E. none of the above

Let $y = e^{\sqrt{x}}$

Then $\ln y = \sqrt{x}$,

and $\frac{1}{y} \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore \quad \frac{dy}{dx} = y \cdot \frac{1}{2\sqrt{x}} = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

14. If $x^2 - y^2 = 1$ then $\frac{dy}{dx} =$

A. $-\frac{y}{x^2}$

B. $\frac{x}{y}$

C. $-\frac{x}{y}$

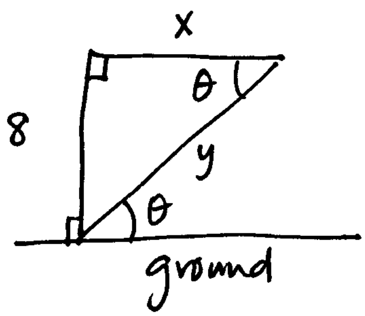
D. $\frac{y}{x}$

E. $\frac{y^2}{x}$

$\longrightarrow 2x - 2y \frac{dy}{dx} = 0$

$\longrightarrow \frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

15. A kite 8 m above the ground flies horizontally at a speed of 2 m/s. Assume our end of the rope is on ground level. When 10 m of string have been let out, the angle between the string and the horizontal is decreasing at a rate of (in rad/s)



Know: $\frac{dx}{dt} = 2$

want: $\frac{d\theta}{dt}$ when $y = 10$ ($\Rightarrow x = 6$)

A. $\frac{1}{25}$

B. $\frac{4}{25}$

C. $\frac{3}{100}$

D. $\frac{1}{10}$

E. $\frac{16}{225}$

$\tan\theta = \frac{8}{x} \rightarrow x = 8\cot\theta$

$\rightarrow \frac{dx}{dt} = -8\csc^2\theta \frac{d\theta}{dt}$

$\rightarrow 2 = -8\left(\frac{10}{8}\right)^2 \frac{d\theta}{dt}$

$\rightarrow \frac{d\theta}{dt} = \frac{2\cdot 8\cdot 8}{-8\cdot 10\cdot 10} = -\frac{4}{25} \rightarrow \theta$ decreasing at $\frac{4}{25}$.

16. The function $f(x) = x^2(6-x)$, $0 \le x \le 6$, attains its absolute maximum when $x =$

A. 0

B. 1

C. 2

D. 4

E. 6

$f(x) = 6x^2 - x^3$

$\rightarrow f'(x) = 12x - 3x^2 = 3x(4-x)$

critical numbers are $x=0$, $x=4$

x	$f(x) = x^2(6-x)$
0	0
4	32 ← abs. max
6	0

17. Suppose $h(x)$ is differentiable and $h'(x) \geq 3$ for all real numbers x; furthermore $h(0) = 0$. Which of the following statements is(are) true?

I. $h(2) \leq 4$

II. $h(2) \geq 6$

III. The equation $h(x) = 0$ has a unique root on $[0, 2]$

A. only I

B. only II

C. only III

D. only I and III

E. only II and III

Mean Value Theorem $\Rightarrow \dfrac{h(2) - h(0)}{2 - 0} = h'(c)$

where $0 < c < 2$.

Since $h'(c) \geq 3$ then $\dfrac{h(2)}{2} = h'(c) \geq 3$

$\rightarrow h(2) \geq 6$ So I is false, II true.

Suppose $h(a) = 0$ where $0 < a \leq 2$. Then by M.V.T. there is a number b, $0 < b < a$ where $\dfrac{h(a) - h(0)}{a - 0} = h'(b)$.

Since $h(a) = h(0) = 0$ then $h'(b) = 0$. But $h'(x) \geq 3$ for all x.

$\therefore$ only $h(0) = 0$ on $[0, 2]$ and III is true.

18. On which one of the following intervals is the function $x^4 - \dfrac{3x^2}{2}$ is concave downward:

A. $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$

B. $\left(0, \frac{\sqrt{3}}{2}\right)$

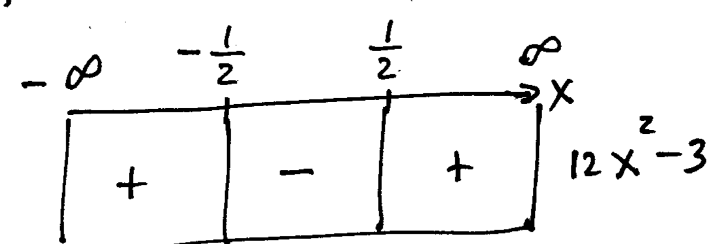
C. $\left(\frac{\sqrt{3}}{2}, \infty\right)$

D. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

E. $\left(\frac{1}{2}, \infty\right)$

$f'(x) = 4x^3 - 3x$ and $f''(x) = 12x^2 - 3$

$f''(x) = 0 \rightarrow x = \pm \frac{1}{2}$



f is concave down

19. $\lim\limits_{x\to 0^+} (1+x)^{2/x} = \lim\limits_{x \to 0^+} e^{\ln(1+x)^{2/x}}$

$= e^{\lim\limits_{x\to 0^+} \ln(1+x)^{2/x}}$

A. 0

B. 1

C. ∞

D. e

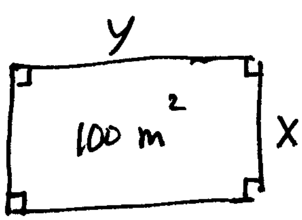
E. e^2

Now, $\lim\limits_{x\to 0^+} \ln(1+x)^{2/x} = \lim\limits_{x\to 0^+} \dfrac{2\ln(1+x)}{x}$

$= \dfrac{0}{0} \overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{2}{1+x}}{1} = 2$

$\therefore \lim\limits_{x\to 0^+} (1+x)^{2/x} = e^2$.

20. A farmer, versed in calculus, wants to fence a rectangular area of 100 m^2, then divide it in two by a fence running along a diagonal. He denotes the length of one side by x. To find the shortest fence needed for this purpose, he will have to find the minimum of the function



$x \to xy = 100$

$\to y = \dfrac{100}{x}$

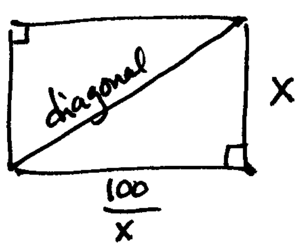
A. $x(100-x)+\sqrt{x^2+(100-x)^2}$

B. $4x(100-x)+\sqrt{x^2+(100-x)^2}$

C. $2x + \dfrac{200}{x} + \sqrt{x^2 + \dfrac{10,000}{x^2}}$

D. $100x + \sqrt{x^2 + \dfrac{100}{x^2}}$

E. $x(100-x) + \sqrt{x^2 + \dfrac{100}{x^2}}$



$x \to$ diagonal $= \sqrt{x^2 + \left(\dfrac{100}{x}\right)^2}$

$\therefore$ length of fencing $= 2(x) + 2\left(\dfrac{100}{x}\right) + \sqrt{x^2 + \left(\dfrac{100}{x}\right)^2}$

21. If $f'(x) = 3\sqrt{x} - 1$ and $f(4) = 0$ then $f(9) =$

A. 7
B. 11
C. 27
D. 33
E. 47

$\rightarrow f(x) = 3 \cdot \frac{2}{3} \cdot x^{3/2} - x + C$

$f(4) = 0 = 2 \cdot 4^{3/2} - 4 + C$

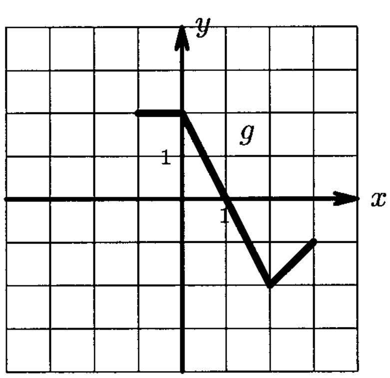
$\rightarrow 0 = 16 - 4 + C$

$\rightarrow C = -12$

$\therefore f(x) = 2x^{3/2} - x - 12$

and $f(9) = 2 \cdot 9^{3/2} - 9 - 12 = 33$

22. If g is given by its graph, $\int_{-1}^{3} g(x)dx =$



A. $\frac{1}{2}$
B. $-\frac{1}{2}$
C. 0
D. 1
E. -1

$\int_{-1}^{3} g(x)\,dx = \int_{-1}^{0} g(x)\,dx + \int_{0}^{1} g(x)\,dx + \int_{1}^{2} g(x)\,dx + \int_{2}^{3} g(x)\,dx$

$= 2 + 1 + (-1) + (-1.5)$

$= 0.5$

23. $\int_{-\frac{1}{2}}^{0} (2x+1)^8 dx =$

Let $u = 2x+1$. Then $du = 2\,dx$,
and $dx = \frac{1}{2}du$.
Also $u(-\frac{1}{2}) = 2(-\frac{1}{2}) + 1 = 0$,
and $u(0) = 2(0)+1 = 1$.

$\therefore \int_{-\frac{1}{2}}^{0} (2x+1)^8 dx = \int_0^1 u^8 \cdot \frac{1}{2} du$

$= \frac{1}{2} \cdot \frac{1}{9} \cdot u^9 \Big|_0^1$

$= \frac{1}{18}(1^9 - 0^9) = \frac{1}{18}$

A. 0
B. $\frac{1}{18}$
C. $-\frac{1}{16}$
D. $\frac{1}{8}$
E. $-\frac{1}{4}$

24. $\int_0^{\frac{\pi}{3}} 2\cos u\,du =$ $2\sin u \Big|_0^{\pi/3}$

$= 2\sin \pi/3 - 2\sin 0$

$= 2 \cdot \frac{\sqrt{3}}{2} - 0$

$= \sqrt{3}$.

A. 1
B. -1
C. $\sqrt{3}$
D. $1-\sqrt{3}$
E. 2

25. If $h(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt$ then $h'(\ln 2) =$

A. 2

B. e^2

C. $\frac{1}{\sqrt{\ln 2}}$

D. $e^{\sqrt{2}}$

E. $2e\ln 2$

$$h'(x) = e^{(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = \frac{e^x}{2\sqrt{x}}$$

$$h'(\ln 2) = \frac{e^{\ln 2}}{2\sqrt{\ln 2}}$$

$$= \frac{2}{2\sqrt{\ln 2}}$$

$$= \frac{1}{\sqrt{\ln 2}}$$