MA 162 FINAL EXAM

May 1, 2002

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Name

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Student ID Number

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Lecturer

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Recitation Instructor

Instructions:

1. This package contains 25 problems, each worth 8 points, for a total of 200 points.
2. Please supply <u>all</u> information requested above and on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes or calculator, please.
5. Some trigonometric formulas:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!} \qquad (1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\sin 2x}{2}$$

$$M_y = \int_a^b x(f(x)-g(x))dx \qquad M_x = \frac{1}{2}\int_a^b (f(x))^2 - (g(x))^2 dx$$

MA 162 FINAL EXAM

SPRING, 2002

1. If θ is the angle between $\vec{u} = -\vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = \vec{i} - 5\vec{j} + \vec{k}$, find $\cos\theta$.

A. $-\frac{4}{3\sqrt{3}}$

B. $-\frac{2}{3}$

C. $-\frac{5}{9}$

D. $-\frac{7}{9}$

E. $-\frac{5}{3\sqrt{3}}$

2. Find a vector of length one that is perpendicular to both $\vec{v} = 3\vec{i} - 2\vec{j} + 2\vec{k}$ and $\vec{w} = -\vec{i} + 2\vec{j} + 2\vec{k}$.

A. $2\vec{i} + 2\vec{j} - \vec{k}$

B. $\frac{2}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} + \frac{1}{\sqrt{6}}\vec{k}$

C. $\frac{2}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$

D. $\frac{1}{\sqrt{5}}\vec{j} - \frac{2}{\sqrt{5}}\vec{k}$

E. $\frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$

3. Compute $\int_1^{e^2} \ln x \, dx$

A. $e^2 + 1$

B. $2e^2 - 1$

C. $e^2 - 1$

D. $2e^2 + 1$

E. e^2

4. Compute $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. 1

E. $\frac{1}{3}$

5. Which integral is obtained if one applies a suitable trigonometric substitution to $\int \frac{\sqrt{9+4x^2}}{x}\,dx$

A. $\int \frac{2\sec\theta}{\tan\theta}$

B. $\int \frac{3\sec^3\theta}{\tan\theta}$

C. $\int \frac{3}{2}\sec\theta\,d\theta$

D. $\int 3\tan^2\theta\,d\theta$

E. $\int \frac{2}{3}\sec\theta\tan\theta\,d\theta$

6. Compute $\int_2^4 \frac{dx}{x^2-1}$.

A. $\frac{1}{2}\ln 5 - \frac{1}{2}\ln 3$

B. $\frac{1}{2}\ln 5 + \frac{1}{2}\ln 3$

C. $\frac{1}{2}\ln 3$

D. $\ln 3 - \frac{1}{2}\ln 5$

E. $\ln 5 - \ln 3$

7. Compute the improper integral $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 2

D. $\frac{\pi}{8}$

E. integral is divergent

8. Compute the area of the surface of revolution obtained by rotating the curve $y = 2\sqrt{x}$, $0 \leq x \leq 3$ about the x-axis.

A. $\frac{8\pi}{3}(3^{\frac{3}{2}} - 1)$

B. $\frac{4\pi}{3}(2\sqrt{2}-1)$

C. $\frac{56\pi}{3}$

D. $\frac{8\pi}{3}(2\sqrt{2}-1)$

E. $\frac{40\pi}{3}$

9. If D is the region bounded by $y = x$ and $y = x^2 - x$, find the x-coordinate of the centroid of D.

A. 1

B. $\frac{5}{4}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

E. $\frac{5}{4}$

10. Let R be the region in the first quadrant bounded above by $y = 2 - x^2$ and below by $y = x$. Compute the volume of the solid obtained by rotating R about the y-axis.

A. $\frac{7\pi}{6}$

B. $\frac{28\pi}{15}$

C. $\frac{13\pi}{5}$

D. $\frac{13\pi}{10}$

E. $\frac{5\pi}{6}$

11. If it takes a force of 5 pounds to stretch a spring 2 feet, how much work is required to stretch the spring an additional foot?

A. 5 ft-lbs

B. $\frac{15}{2}$ ft-lbs

C. $\frac{45}{4}$ ft-lbs

D. $\frac{25}{2}$ ft-lbs

E. $\frac{25}{4}$ ft-lbs

12. Let R be the region lying under the graph of $y = \sin x$ between 0 and π. What is the volume of the solid obtained by rotating R around the x-axis?

A. $\frac{\pi^2}{2}$

B. $\frac{\pi}{2}$

C. $\pi^2 - \frac{\pi}{2}$

D. $\frac{\pi^2}{4}$

E. $\frac{3\pi}{2}$

13. Which of the following statements is true for $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

A. It converges because $\lim_{n\to\infty} \frac{1}{n(\ln n)^2} = 0$

B. It converges by the Ratio Test

C. It diverges by the Ratio Test

D. It converges by the Integral Test

E. It diverges by the Integral Test

14. $\sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}} =$

A. 1

B. $\frac{8}{3}$

C. 2

D. 3

E. $\frac{4}{3}$

15. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+n}}$

(II) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(III) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$

A. I only

B. II only

C. III only

D. I and II only

E. II and III only

16. Which of the following series converge conditionally?

I. $\sum_{n=1}^{\infty} (-1)^n \cos \frac{1}{n}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$

A. I only

B. II only

C. III only

D. I and II only

E. II and III only

17. The radius of convergence of the series $\sum_{n=1}^{\infty} \left(\frac{3n-1}{2n+1}\right)^n x^n$ is

A. 2

B. $\frac{1}{3}$

C. $\frac{3}{2}$

D. $\frac{2}{3}$

E. 1

18. $\int_0^1 \frac{e^x - 1}{x}\,dx =$

A. $\sum_{n=1}^{\infty} \frac{1}{n!n}$

B. $\sum_{n=1}^{\infty} \frac{1}{(n+1)!}$

C. $\sum_{n=1}^{\infty} \frac{1}{(n+1)!n}$

D. $\sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

E. $\sum_{n=1}^{\infty} \frac{1}{(n-1)!n}$

19. The Maclaurin series for $\dfrac{x}{1+2x^2}$ is given by

A. $\sum_{n=0}^{\infty} 2^n x^{2n}$

B. $\sum_{n=0}^{\infty} 2^n x^{2n+1}$

C. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n}$

D. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{n+1}$

E. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1}$

20. If $\sin x$ is expanded as a power series of the form $\sum_{n=0}^{\infty} c_n \left(x - \frac{\pi}{3}\right)^n$, then $c_3 =$

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{12}$

D. $-\frac{1}{12}$

E. $-\frac{\sqrt{3}}{12}$

21. Given a curve with parametric equations $x = te^t$, $y = 1 + \sqrt{t+1}$, find the slope of the tangent line to the curve at $t = 0$.

A. $2\sqrt{2}$

B. 2

C. $\frac{1}{2}$

D. 0

E. $\frac{\sqrt{2}}{2}$

22. The area of the region that lies inside $r = 4\sin\theta$ and outside $r = 2$ is given by

A. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}((4\sin\theta)^2 - 4)d\theta$

B. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{2}}((4\sin\theta)^2 - 4)d\theta$

C. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}((4\sin\theta)^2 - 4)d\theta$

D. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}((4\sin\theta)^2 - 4)d\theta$

E. $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}((4\sin\theta)^2 - 4)d\theta$

23. Find a focus of the conic section whose equation is $2x^2 + y - 8x + 9 = 0$.

A. $\left(2, \frac{7}{8}\right)$

B. $\left(2, -\frac{7}{8}\right)$

C. $\left(2, \frac{9}{8}\right)$

D. $\left(2, -\frac{9}{8}\right)$

E. $\left(2, -\frac{1}{2}\right)$

24. An ellipse has foci at $(\pm 1, 2)$ and the length of its major axis is 6. Its equation is

A. $\frac{x^2}{9} + \frac{(y-2)^2}{8} = 1$

B. $\frac{x^2}{4} + \frac{(y-2)^2}{9} = 1$

C. $\frac{x^2}{8} + \frac{(y-2)^2}{9} = 1$

D. $\frac{x^2}{9} + \frac{(y-2)^2}{4} = 1$

E. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$

25. Find the length of the polar curve $r = \sin^3\left(\frac{\theta}{3}\right)$, $0 \leq \theta \leq \pi$.

A. $\pi - \frac{3\sqrt{3}}{4}$

B. $\frac{\pi}{2} - \frac{3\sqrt{3}}{8}$

C. $\frac{\pi}{2} - \frac{3\sqrt{3}}{4}$

D. $\pi - \frac{1}{2}$

E. $\frac{\pi}{2} - \frac{1}{4}$