MA 162 EXAM **October 4, 2001**

Name _____ Student ID number _____

Lecturer _____ Recitation Instructor _____

Instructions:

1. This package contains 13 problems, each worth 8 points, for a total of 104 points.
2. Please supply all information requested above and on the mark–sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly.
4. No books, notes, or calculator, please.
5. Some trigonometric formulas:

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2},$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2},$$

$$\sin \alpha \cos \alpha = \frac{\sin 2\alpha}{2}.$$

1. $\int_1^3 \frac{\ln x}{x^2}\,dx =$

A. $(\ln 3) - \frac{1}{3}$

B. $3 - \frac{\ln 3}{3}$

C. $\frac{2}{3} - \frac{\ln 3}{3}$

D. $\frac{2\ln 3}{3}$

E. $3(\ln 3) - \frac{1}{3}$

2. $\int \cos^2 x \sin^3 x\,dx =$

A. $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$

B. $\frac{\cos^2 x}{2} - \frac{\cos^4 x}{4} + C$

C. $\frac{\sin^2 x}{2} - \frac{\sin^4 x}{4} + C$

D. $\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$

E. None of the above

3. In the course of computing $\int_0^{\pi/3} \sec^4 x \, dx$ which substitution should be used?

A. $u = \sin x$

B. $u = \cos x$

C. $u = \tan x$

D. $u = \sec x$

E. $u = \sec^{-1} x$

4. If $\sin t = \frac{x}{2}$ $(0 \leq t \leq \frac{\pi}{2})$ then $\tan t =$

A. $\sqrt{1 - \frac{4}{x^2}}$

B. $\frac{x}{\sqrt{1 - x^2}}$

C. $\frac{x}{\sqrt{1 + 4x^2}}$

D. $\frac{x}{\sqrt{4 - x^2}}$

E. $\frac{\sqrt{1 - 4x^2}}{\sqrt{1 + 4x^2}}$

5. The integral $\int \frac{dx}{\sqrt{4x^2+9}}$ can be computed by means of a certain substitution. Which of the following integrals would be obtained after this substitution?

A. $\int \frac{\tan\theta}{2}\,d\theta$

B. $\int \frac{2\sec^2\theta}{3}\,d\theta$

C. $\int \frac{\sec\theta}{2}\,d\theta$

D. $\int \frac{\sec\theta\tan\theta}{3}\,d\theta$

E. $\int \frac{\tan^2\theta}{2}\,d\theta$

6. Which substitution should be used to compute $\int \frac{dx}{\sqrt{3-2x-x^2}}$?

A. $x = -1 + 2\sin t$

B. $x = 1 - 2\tan t$

C. $x = 1 - \sin t$

D. $x = 2\sin t$

E. $x = 1 + \tan t$

7. The function $\dfrac{4}{x^2(x+2)}$ decomposes into partial fractions as follows:

A. $\dfrac{2}{x} - \dfrac{3}{x^2} + \dfrac{1}{x+2}$

B. $\dfrac{1}{2x} - \dfrac{4}{x^2} - \dfrac{2}{x+2}$

C. $-\dfrac{1}{x} + \dfrac{2}{x^2} + \dfrac{1}{x+2}$

D. $\dfrac{1}{x} + \dfrac{3}{x^2} + \dfrac{2}{x+2}$

E. none of the above

8. $\displaystyle\int_0^1 \frac{x+2}{x^2+1}\,dx =$

A. π

B. $\dfrac{\pi + \ln 2}{2}$

C. $1 + \ln 3 - \ln 2$

D. $-\dfrac{1}{2}$

E. $1 + \dfrac{\pi}{2}$

9. Which among the following integrals is/are improper?

I. $\int_0^1 \frac{dx}{2x}$ II. $\int_0^2 \frac{dx}{1+x}$ III. $\int_0^\infty 2^{-x}dx$

A. Only II
B. Only III
C. Only I and II
D. Only I and III
E. All three

10. Which is true?

I. $\int_1^\infty \frac{2}{x^2}\,dx = 2.$

II. $\int_1^\infty \frac{(1+x)^{1/3}}{x}\,dx$ converges by the comparison test.

III. $\int_1^\infty \frac{(1+x)^{1/3}}{x}\,dx$ diverges by the comparison test.

A. Only I
B. Only II
C. Only I and II
D. Only II and III
E. Only I and III

11. Find the length of the curve $y = f(x)$, $0 \leq x \leq 2$, given that $f'(x) = \sqrt{4x^2 + 4x}$.

A. 2

B. 3

C. 4

D. 5

E. 6

12. The area of the surface obtained by rotating the curve $y = \frac{x^3}{3}$, $0 \leq x \leq 1$, about the x axis is

A. $\frac{\pi(2\sqrt{2}-1)}{9}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi^2\sqrt{2}}{9}$

D. 3π

E. $\frac{4\pi}{3} - \frac{1}{3}$

13. A mass of 2 grams is located at the point $(2, 0)$. What mass should be placed at the point $(8, 0)$ so that the center of mass of the system is at $(4, 0)$?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. 2

E. 3