NAME GRADING KEY

STUDENT ID _____

RECITATION INSTRUCTOR _____

RECITATION TIME _____

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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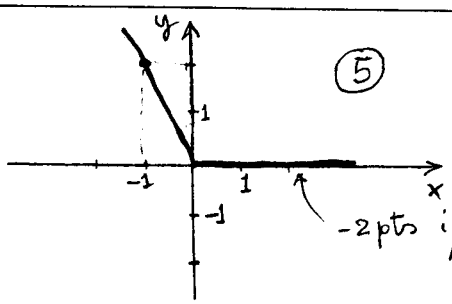
(8) 1. Let $f(x) = |x| - x$.

(a) Sketch the graph of f.



⑤

-2 pts if missing

Complete the following:

(b) f is not differentiable at $x =$ [0] ①

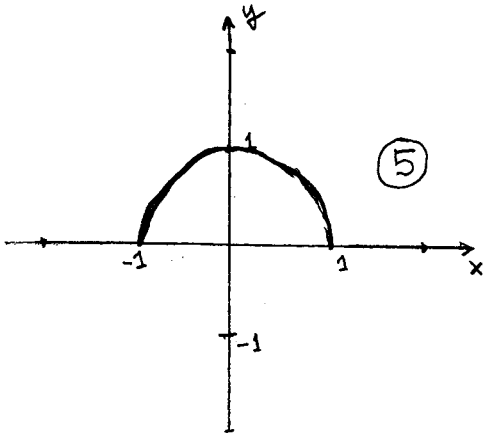
(c) $f'(2) =$ [0] ①

(d) $f'(-1) =$ [-2] ①

[8]

(7) 2. Let $g(x) = \sqrt{1 - x^2}$.

(a) Sketch the graph of g.



⑤

(b) The domain of g is [-1, 1] or $-1 \le x \le 1$ ①

(c) The range of g is [0, 1] or $0 \le y \le 1$ ①

[7]

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(8) 3. Find all solutions of the equation $\sin 2x = \cos x$ for x in $[0, 2\pi)$.

$2\sin x\cos x = \cos x$ (2)

$(2\sin x - 1)\cos x = 0$ (2)

$2\sin x - 1 = 0 \longrightarrow \sin x = \frac{1}{2} \longrightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\cos x = 0 \longrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

(1) (1) (1) (1)

$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ [8]

(6) 4. Evaluate the following:

(a) $\ln|\ln\frac{1}{e}| = \ln|-\ln e| = \ln 1 = 0$ — 0 [2]

(b) $\sin(\pi e^{-\ln 2}) = \sin\left(\frac{\pi}{e^{\ln 2}}\right) = \sin\frac{\pi}{2} = 1$ — 1 [2]

(c) $\cos(-\pi \ln 1) = \cos 0 = 1$ — 1 [2]

(12) 5. Find each of the following. Fill in the boxes below with a finite number, or one of the symbols: ∞, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

2 pts for each correct answer

$\lim_{x\to 0} \frac{\sin 3x}{5x} =$ $\frac{3}{5}$

$\lim_{t\to 5^+} \frac{|t-5|}{5-t} =$ -1

$\lim_{x\to 1^-} \ln(1-x) =$ $-\infty$

$\lim_{x\to 0} x\cos\frac{1}{x} =$ 0

$\lim_{z\to 3} \frac{2}{(z-3)^2} =$ ∞

$\lim_{x\to 0} \frac{e^{2x}-1}{e^x-1} =$ 2

[12]

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(10) 6. Use the definition of the derivative of f at a: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find $f'(-2)$ if $f(x) = \frac{1}{x}$. Show all steps.

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \to -2} \frac{\frac{1}{x} - \frac{1}{-2}}{x + 2} \quad (4)$$

$$= \lim_{x \to -2} \frac{x+2}{2x(x+2)} = \underbrace{\lim_{x \to -2} \frac{1}{2x}}_{(4)} = -\frac{1}{4} \quad (2)$$

-2pts if $\lim_{x \to -2}$ is missing anywhere

10

(5) 7. Find a if $f(x) = -2x^2$ and $f'(a) = 12$.

$f'(x) = -4x$ $\quad -4a = 12$ (4)

$a = -3$ (1)

-3

5

(8) 8. Show that the equation $x^3 - x - 5 = 0$ has at least one solution. State the name of the theorem you are using.

$f(x) = x^3 - x - 5$ (2)

$f(0) = -5$ (2)

$f(2) = 1$ (2)

f is continuous on $[0, 2]$

$f(0) < 0 < f(2)$

Intermediate value theorem (2)

8

(7) 9. If $f(t) = \frac{-2t}{\sin t}$, find $f'\left(\frac{\pi}{2}\right)$.

$$f'(t) = \frac{\sin t(-2) - (-2t)\cos t}{\sin^2 t} \quad (5) \qquad f'\left(\frac{\pi}{2}\right) = -2 \quad (2)$$

-2

7

(9) 10. Find an equation of the line tangent to the graph of $f(x) = \frac{1}{(1+x)^2}$ at the point $(0, 1)$.

$f'(x) = -\frac{2}{(1+x)^3}$ (4)

$f'(0) = -2$ (2)

$y - 1 = (-2)(x - 0)$ (3)

or

$y = -2x + 1$

-1 pt for minor numerical error

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(20) 11. Find the derivatives of the following functions. (It is not necessary to simplify.)

(a) $f(t) = 3\cos(\pi t^2)$

NPC

$f'(t) = 3(-\sin(\pi t^2))\, 2\pi t$ or $-6\pi t \sin(\pi t^2)$ [4]

(b) $f(x) = \tan^5 x$.

$5\tan^4 x \sec^2 x$ [4]

(c) $f(x) = \sqrt{4x - \sqrt{x}}$.

$f'(x) = \frac{1}{2}\frac{1}{\sqrt{4x-\sqrt{x}}}\left(4 - \frac{1}{2\sqrt{x}}\right)$ [4]

(d) $f(x) = e^{-5x}\sin 2x$.

$f'(x) = e^{-5x}\cos 2x \cdot 2 + (-5)e^{-5x}\sin 2x$ or $e^{-5x}(2\cos 2x - 5\sin 2x)$ [4]

(e) $g(t) = \ln(\sin e^t)$.

$g'(t) = \frac{1}{\sin e^t}(\cos e^t)\, e^t$ or $e^t \cot e^t$ [4]