NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(10) 1. Use the <u>second derivative test</u> to determine the relative maximum and minimum values (if any) of the function $g(x) = x^4 - 8x^2 + 1$. Give your answers in the form: "$g(a) = b$ is a rel. max (or min)."

$g'(x) = 4x^3 - 16x$ ①

$g'(x) = 0$: $4x^3 - 16x = 0 \longrightarrow 4x(x+2)(x-2) = 0 \longrightarrow x = \overbrace{-2,\ 0,\ 2}^{②}$

−1 pt for each wrong or missing cr. pt.

$g''(x) = 12x^2 - 16$ ①

① $g''(-2) = 12\cdot4 - 16 > 0$ ∴ $g(-2) = 16 - 32 + 1 = -15$ is a rel. min

① $g''(0) = -16 < 0$ ∴ $g(0) = 1$ is a rel. max.

① $g''(2) = 12\cdot4 - 16 > 0$ ∴ $g(2) = 16 - 32 + 1 = -15$ is a rel. min

① $g(-2) = -15$ is a rel. min
① $g(0) = 1$ is a rel. max
① $g(2) = -15$ is a rel. min

10

(6) 2. (a) $\lim_{x\to-\infty} e^{-\frac{1}{x}} = \boxed{1}$ ③ (b) $\lim_{x\to\infty} \frac{1}{\ln x} = \boxed{0}$ ③

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Answers only. No reason necessary.

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(10) 3. Let $f(x) = 3x^5 - 5x^3$.

(a) Find a point c such that $f'(c) = f''(c) = 0$.

$f'(x) = 15x^4 - 15x^2$ ① $f''(x) = 60x^3 - 30x$ ①

$f'(c) = 0 : 15c^4 - 15c^2 = 0 \rightarrow c^2(c^2-1) = 0 \rightarrow c = 0, -1, 1$

$f''(c) = 0 : 60c^3 - 30c = 0 \quad c(2c^2-1) = 0 \rightarrow c = 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

③ $\boxed{c = 0}$

(b) Circle the letter of the correct statement and give the reason for your answer.

A. f has a relative extreme value at c.

[✓ B.] f does not have a relative extreme value at c.

both ⑤

Reason:

$f'(x) = 15x^2(x^2-1) = 15x^2(x+1)(x-1)$

$f'(x)$ + + + + + | − − − | − − − | + + + . . .

-1 0 1

or f is decreasing in $[-1, 1]$ or f is decreasing before and after 0.

or f' does not change sign at 0.

$\boxed{10}$

(12) 4. Let $f(x) = \dfrac{x}{x^2-1}$.

(a) Find the inflection points (x, y) (if any) of the graph of f.

$$f'(x) = \frac{(x^2-1)1 - x \cdot 2x}{(x^2-1)^2} = -\frac{x^2+1}{(x^2-1)^2} \quad ②$$

$$f''(x) = -\frac{(x^2-1)^2 2x - (x^2+1)2(x^2-1)2x}{(x^2-1)^4} \quad ②$$

$$= -\frac{2x^3 - 2x - 4x^3 - 4x}{(x^2-1)^3} = -\frac{-2x^3 - 6x}{(x^2-1)^3}$$

$$= \frac{x(2x^2+6)}{(x^2-1)^2} \qquad f''(x) = 0 \rightarrow x = 0$$

②

$f''(0) = 0$ and $f''(x)$ changes sign at $x=0$ $\boxed{(0, 0)}$

(b) Find the vertical and horizontal asymptotes (if any) of the graph of f.

$$f(x) = \frac{x}{(x+1)(x-1)}$$

Vertical asymptote(s): ②

$x \overset{②}{=} -1$ and $x = -1$

Horizontal asymptote(s):

$y = 0$ ②

$\boxed{12}$

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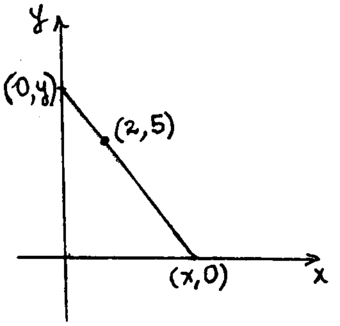
(12) 5. A right triangle is formed by the coordinate axes and the line through the point $(2,5)$, as shown in the figure below. Find the value of x for which the area of the triangle is minimum.



$A = \frac{1}{2} x y \qquad \frac{y}{x} = \frac{5}{x-2} \rightarrow y = 5\frac{x}{x-2}$ ⑤

$A = \frac{5}{2}\frac{x^2}{x-2}$ ② $, \quad 2 < x < \infty$

$\frac{dA}{dx} = \frac{5}{2}\frac{(x-2)2x - x^2\cdot 1}{(x-2)^2} = \frac{5}{2}\frac{2x^2-4x-x^2}{(x-2)^2}$

$= \frac{5}{2}\frac{x(x-4)}{(x-2)^2}$ ②

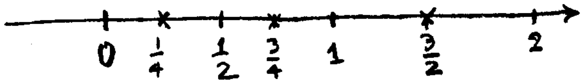
$\frac{dA}{dx} = 0 \longrightarrow \cancel{x=0}, \quad x = 4$ ③

$\frac{dA}{dx}$ ——$\underset{2}{|}$- - -$\underset{4}{|}$+ + +—— $\therefore$ A is min at $x=4$

$\boxed{x = 4}$

12

(6) 6. Approximate $\int_0^2 \sin(\pi x)dx$ using the midpoint Riemann sum with the partition $P = \{0, \frac{1}{2}, 1, 2\}$.



$f(x) = \sin \pi x$

$\sin\frac{\pi}{4}\cdot\frac{1}{2} + \sin\frac{3\pi}{4}\cdot\frac{1}{2} + \sin\frac{3\pi}{2}\cdot 1$

$= \underbrace{\frac{\sqrt{2}}{2}\cdot\frac{1}{2}}_{②} + \underbrace{\frac{\sqrt{2}}{2}\cdot\frac{1}{2}}_{②} \underbrace{-1}_{②} = \frac{\sqrt{2}}{2} - 1$

$\boxed{\frac{\sqrt{2}}{2} - 1}$

6

(8) 7. Find the derivative of the function

$$G(y) = \int_y^{y^2} (1+t^2)^{\frac{1}{2}} dt.$$

$G(y) = \int_y^0 (1+t^2)^{\frac{1}{2}}dt + \int_0^{y^2} (1+t^2)^{\frac{1}{2}}dt$ ②

$= -\int_0^y (1+t^2)^{\frac{1}{2}}dt + \int_0^{y^2} (1+t^2)^{\frac{1}{2}}dt$

$G'(y) = -(1+y^2)^{\frac{1}{2}} + (1+y^4)^{1/2}2y$

③ ③

$\boxed{G'(y) = -(1+y^2)^{\frac{1}{2}} + 2y(1+y^4)^{1/2}}$

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(8) 8. $\int_0^\pi (\sin x - 2e^x)dx = \left(\overset{②}{-\cos x} - \overset{②}{2e^x}\right)\Big|_0^\pi$

$= (-\cos\pi - 2e^\pi) - (-\cos 0 - 2)$

$= 1 - 2e^\pi + 1 + 2$

④ $\boxed{4-2e^\pi}$ $\boxed{8}$

−1pt for minor numerical error or for not giving values of cos π, cos 0.

(12) 9. (a) $\int \frac{\ln z}{z}dz = \int u\,du = \frac{u^2}{2} + C = \frac{(\ln z)^2}{2} + C$

$u = \ln z$

$du = \frac{1}{z}dz$

⑥ $\boxed{\frac{1}{2}(\ln z)^2 + C}$

−1pt if +C is missing

(b) $\int x\sqrt{x+2}\,dx = \int (u-2)u^{1/2}du = \int (u^{3/2} - 2u^{1/2})du =$

$u = x+2 \quad du = dx$

$x = u-2$

$= \frac{u^{5/2}}{\frac{5}{2}} - 2\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

⑥ $\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$ $\boxed{12}$

(8) 10. Find the area of the region between the graph of the function $f(x) = \frac{x}{x^2+1}$ and the x-axis on the interval $[0,1]$.

$A = \int_0^1 \frac{x}{x^2+1}dx$ ④ $= \frac{1}{2}\int_1^2 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_1^2 = \frac{1}{2}\ln 2$ ④

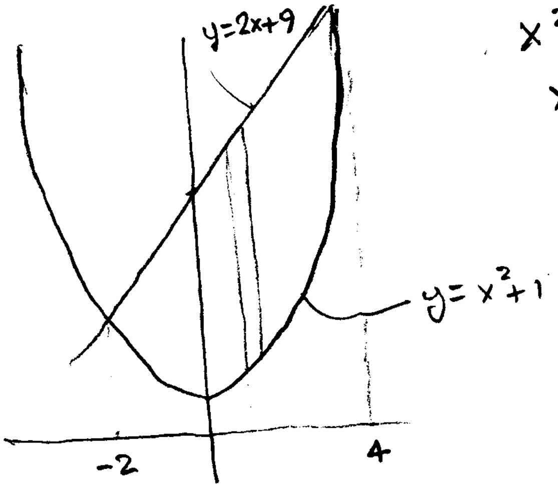
$u = x^2+1 \quad du = 2x\,dx$

$x = 0 \to u = 1 \quad x = 1 \to u = 2$

−1pt for missing dx

$\boxed{\frac{1}{2}\ln 2}$ $\boxed{8}$

(8) 11. Set up a definite integral for the area of the region enclosed by the graphs of the functions $f(x) = x^2 + 1$ and $g(x) = 2x + 9$. Do not evaluate the integral.



$x^2 + 1 = 2x + 9$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

0 credit for problem if more than one error in box

$\boxed{A = \int_{\underset{②}{-2}}^{\overset{②}{4}} \overbrace{\left[(2x+9) - (x^2+1)\right]}^{③}\, \overset{①}{dx}}$