Using Linear Combinations

Express the vectors in Exercises 19–24 in the form $a\mathbf{i} + b\mathbf{j}$ and sketch them as arrows in the coordinate plane beginning at the origin.

19. $\overrightarrow{P_1P_2}$ if P_1 is the point (5, 7) and P_2 is the point (2, 9)
20. $\overrightarrow{P_1P_2}$ if P_1 is the point (1, 2) and P_2 is the point (−3, 5)
21. $\overrightarrow{AB}$ if A is the point (−5, 3) and B is the point (−10, 8)
22. $\overrightarrow{AB}$ if A is the point (−7, −8) and B is the point (6, 11)
23. $\overrightarrow{P_1P_2}$ if P_1 is the point (1, 3) and P_2 is the point (2, −1)
24. $\overrightarrow{P_3P_4}$ if P_3 is the point (1, 3) and P_4 is the midpoint of the line segment P_1P_2 joining $P_1(2, -1)$ and $P_2(-4, 3)$

Unit Vectors

Sketch the vectors in Exercises 25–28 and express each vector in the form $a\mathbf{i} + b\mathbf{j}$.

25. The unit vectors $\mathbf{u} = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$ for $\theta = \pi/6$ and $\theta = 2\pi/3$. Include the circle $x^2 + y^2 = 1$ in your sketch.
26. The unit vectors $\mathbf{u} = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$ for $\theta = -\pi/4$ and $\theta = -3\pi/4$. Include the circle $x^2 + y^2 = 1$ in your sketch.
27. The unit vector obtained by rotating $\mathbf{j}$ counterclockwise $3\pi/4$ rad about the origin
28. The unit vector obtained by rotating $\mathbf{j}$ clockwise $2\pi/3$ rad about the origin

In Exercises 29–32, find a unit vector in the direction of the given vector.

29. $\langle 3, 4\rangle$
30. $\langle 4, -3\rangle$
31. $\langle -15, 8\rangle$
32. $\langle -5, -2\rangle$

For the vectors in Exercises 33 and 34, find unit vectors $\mathbf{u} = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$ in the same direction.

33. $6\mathbf{i} - 8\mathbf{j}$
34. $-\mathbf{i} + 3\mathbf{j}$

Length and Direction

In Exercises 35 and 36, express each vector as a product of its length and direction.

35. $5\mathbf{i} + 12\mathbf{j}$
36. $2\mathbf{i} - 3\mathbf{j}$
37. Find the unit vectors that are parallel to the vector $3\mathbf{i} - 4\mathbf{j}$ (two vectors in all).
38. Find a vector of length 2 whose direction is the opposite of the direction of the vector $-\mathbf{i} + 2\mathbf{j}$. How many such vectors are there?

Tangent and Normal Vectors

In Exercises 39–42, find the unit vectors that are tangent and normal to the curve at the given point (four vectors in all). Then sketch the vectors and curve together.

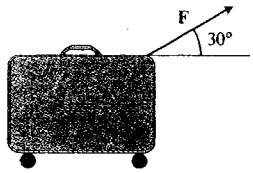
39. $y = x^2$, $(2, 4)$
40. $x^2 + 2y^2 = 6$, $(2, 1)$
41. $y = \tan^{-1} x$, $(1, \pi/4)$
42. $y = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $(0, 1)$

In Exercises 43–46, find the unit vectors that are tangent and normal to the curve at the given point (four vectors in all).

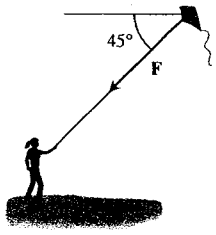
43. $3x^2 + 8xy + 2y^2 - 3 = 0$, $(1, 0)$
44. $x^2 - 6xy + 8y^2 - 2x - 1 = 0$, $(1, 1)$
45. $y = \int_0^x \sqrt{3 + t^4}\, dt$, $(0, 0)$
46. $y = \int_e^x \ln(\ln t)\, dt$, $(e, 0)$

Theory and Applications

47. *Linear combination* Let $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$, and $\mathbf{w} = \mathbf{i} - \mathbf{j}$. Find scalars a and b such that $\mathbf{u} = a\mathbf{v} + b\mathbf{w}$.
48. *Linear combination* Let $\mathbf{u} = \mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$, and $\mathbf{w} = \mathbf{i} + \mathbf{j}$. Write $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$, where $\mathbf{u}_1$ is parallel to $\mathbf{v}$ and $\mathbf{u}_2$ is parallel to $\mathbf{w}$. (See Exercise 47.)
49. *Force vector* You are pulling on a suitcase with a force $\mathbf{F}$ (pictured here) whose magnitude is $|\mathbf{F}| = 10$ lb. Find the $\mathbf{i}$- and $\mathbf{j}$-components of $\mathbf{F}$.



50. *Force vector* A kite string exerts a 12 lb pull ($|\mathbf{F}| = 12$) on a kite and makes a 45° angle with the horizontal. Find the horizontal and vertical components of $\mathbf{F}$.



51. *Velocity* An airplane is flying in the direction 25° west of north at 800 km/h. Find the component form of the velocity of the airplane, assuming that the positive x-axis represents due east and the positive y-axis represents due north.
52. *Velocity* An airplane is flying in the direction 10° east of south at 600 km/h. Find the component form of the velocity of the airplane, assuming that the positive x-axis represents due east and the positive y-axis represents due north.

The vector **u** is tangent to the curve at (1, 1) because it has the same direction as **v**. Of course,

$$-\mathbf{u} = -\frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j},$$

which points in the opposite direction, is also tangent to the curve at (1, 1). Without some additional requirement (such as specifying the direction of motion), there is no reason to prefer one of these vectors to the other.

To find unit vectors normal to the curve at (1, 1), we look for unit vectors whose slopes are the negative reciprocal of the slope of **u**. This is quickly done by interchanging the scalar components of **u** and changing the sign of one of them. We obtain

$$\mathbf{n} = -\frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{j} \quad \text{and} \quad -\mathbf{n} = \frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{j}.$$

Again, either one will do. The vectors have opposite directions but both are normal to the curve at (1, 1). (See Figure 9.13.)

EXERCISES 9.1

Component Form

In Exercises 1–8, let $\mathbf{u} = \langle 3, -2\rangle$ and $\mathbf{v} = \langle -2, 5\rangle$. Find the **(a)** component form and **(b)** magnitude (length) of the vector.

1. $3\mathbf{u}$
2. $-2\mathbf{v}$
3. $\mathbf{u} + \mathbf{v}$
4. $\mathbf{u} - \mathbf{v}$
5. $2\mathbf{u} - 3\mathbf{v}$
6. $-2\mathbf{u} + 5\mathbf{v}$
7. $\frac{3}{5}\mathbf{u} + \frac{4}{5}\mathbf{v}$
8. $-\frac{5}{13}\mathbf{u} + \frac{12}{13}\mathbf{v}$

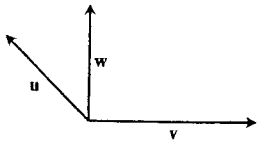
In Exercises 9–16, find the component form of the vector.

9. The vector $\overrightarrow{PQ}$, where $P = (1, 3)$ and $Q = (2, -1)$
10. The vector $\overrightarrow{OP}$ where O is the origin and P is the midpoint of segment RS, where $R = (2, -1)$ and $S = (-4, 3)$
11. The vector from the point $A = (2, 3)$ to the origin
12. The sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$, where $A = (1, -1)$, $B = (2, 0)$, $C = (-1, 3)$, and $D = (-2, 2)$
13. The unit vector that makes an angle $\theta = 2\pi/3$ with the positive x-axis
14. The unit vector that makes an angle $\theta = -3\pi/4$ with the positive x-axis
15. The unit vector obtained by rotating the vector $\langle 0, 1\rangle$ 120° counterclockwise about the origin
16. The unit vector obtained by rotating the vector $\langle 1, 0\rangle$ 135° counterclockwise about the origin

Geometry and Calculation

In Exercises 17 and 18, copy vectors **u**, **v**, and **w** head to tail as needed to sketch the indicated vector.

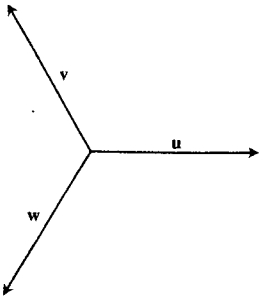
17.



(a) $\mathbf{u} + \mathbf{v}$ (b) $\mathbf{u} + \mathbf{v} + \mathbf{w}$

(c) $\mathbf{u} - \mathbf{v}$ (d) $\mathbf{u} - \mathbf{w}$

18.



(a) $\mathbf{u} - \mathbf{v}$ (b) $\mathbf{u} - \mathbf{v} + \mathbf{w}$

(c) $2\mathbf{u} - \mathbf{v}$ (d) $\mathbf{u} + \mathbf{v} + \mathbf{w}$