MA 220 **Final Exam Practice Problems**

The Table of Integrals (pages 558-559 of the text) may be used. The Table of Integrals will be attached to the final exam.

1. If $f(x) = x^2 - 1$, calculate $f(\frac{1}{2})$ and $\frac{1}{f(2)}$.
   A. $f(\frac{1}{2}) = \frac{1}{3}$; $\frac{1}{f(2)} = -\frac{3}{4}$ B. $f(\frac{1}{2}) = -\frac{3}{4}$; $\frac{1}{f(2)} = \frac{1}{3}$ C. $f(\frac{1}{2}) = -\frac{3}{4}$; $\frac{1}{f(2)} = -\frac{4}{3}$
   D. $f(\frac{1}{2}) = 3$; $\frac{1}{f(2)} = \frac{1}{3}$ E. None of these.

2. Find the slope of the line containing the points $(-2, 4)$ and $(6, -3)$.
   A. 4 B. $-7/8$ C. $1/4$ D. $-8/7$ E. $-1/2$

3. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons T and days d is linear, express T as a function of d.
   A. $T = 5d + 280$ B. $T = -44d + 500$ C. $T = 44d + 60$ D. $T = 60d + 44$ E. $T = 60d + 280$

4. The domain of $f(x) = \dfrac{1}{\sqrt[3]{x-1}}$ is all real numbers x such that
   A. $x \neq 1$ B. $x > 1$ C. $x > 0$ D. $x \neq 0$ E. $-1 < x < 1$

5. If $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 7$ then $f \circ g(-1)) =$
   A. 0 B. 3 C. $\sqrt{7}$ D. 7 E. $\sqrt{3}$

6. $\lim\limits_{x \to 1} \dfrac{x^2 + 4x - 5}{x^2 - 1} =$
   A. ∞ B. 0 C. 3 D. -3 E. 5

7. If $f(x) = \dfrac{2}{x}$, find a simplified form for the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.
   A. $\dfrac{-2}{x^2}$ B. $\dfrac{2}{x+h} - \dfrac{2}{x}$ C. $\dfrac{2}{x(x+h)}$ D. $\dfrac{-2}{x(x+h)}$ E. $\dfrac{-2}{x^2+h}$

8. For what value of a does the function $f(x) = x^2 + ax$ have a relative minimum at $x = 1$.
   A. -2 B. 0 C. 2 D. -1 E. 1

9. The derivative of $\dfrac{x^2+1}{x+5}$ is
   A. $\dfrac{(x+5)2x - (x^2+1)}{(x+5)^2}$ B. $2x$ C. $\dfrac{(x+5)}{(x^2+1)^2} \cdot 2x$
   D. $\dfrac{(x^2+1) + (x+5)2x}{(x+5)^2}$ E. $\dfrac{(x^2+1) - (x+5)2x}{(x+5)^2}$

10. If $y = (3 - x^2)^3$ then $\dfrac{d^2y}{dx^2} =$
    A. $-6x(3-x^2)^2$ B. $24x^2(3-x^2) - 6(3-x^2)^2$ C. $6(3-x^2)$
    D. $24x^2(3-x^2)$ E. None of these.

11. The line tangent to the graph of $f(x) = x - \dfrac{1}{x}$ at $x = 2$ has slope
    A. 5/4 B. 3/4 C. 3/2 D. 0 E. 1/4

12. A total cost function is given by $C(x) = 1000\sqrt{x^2+2}$. Calculate $C'(10)$. Give your answer correct to two decimal places.
    A. 10,099.50 B. 990.15 C. 49.51 D. 99.01 E. 499.15

13. Find all open intervals on which the function $f(x) = 2x^3 - 3x^2 - 12x + 12$ is increasing.
A. $(-1, 2)$ B. $(-\infty, -1)$ C. $(2, \infty)$ D. $(-\infty, -1)$ and $(2, \infty)$ E. $(-1, 2)$ and $(2, \infty)$

14. If the concentration $C(t)$ of a certain drug remaining in the bloodstream t minutes after it is injected is given by $C(t) = t/(5t^2 + 125)$, then the concentration is a maximum when $t =$
A. 25 B. 15 C. 5 D. 10 E. There is no maximum

15. If $f(x) = 2x^4 - 6x^2$ then which one of the following is true?
A. f has a relative max. at $x = \pm\sqrt{3/2}$ and a relative min at $x = 0$.
B. f has a relative max. at $x = 0$ and a relative min. at $x = \pm\sqrt{3/2}$.
C. f has a relative max. at $x = -\sqrt{3/2}$ and a relative min. at $x = \sqrt{3/2}$.
D. f has no relative max. points, but has relative min. at $x = \pm\sqrt{3/2}$.
E. None of these.

16. The derivative of a function f is $f'(x) = x^2 - \dfrac{8}{x}$. Then at $x = 2$, f has
A. an inflection point B. a relative maximum C. a vertical tangent
D. a vertical asymptote E. a relative minimum

17. If $f(x) = \frac{1}{3}x^3 - 9x + 2$. Then on the closed interval $0 \le x \le 4$,
A. f has an absolute max. at $x = 3$ and an absolute min. at $x = 0$.
B. f has an absolute max. at $x = 4$ and an absolute min. at $x = 3$.
C. f has an absolute max. at $x = 0$ and an absolute min. at $x = 4$.
D. f has an absolute max. at $x = 0$ and an absolute min. at $x = 3$.
E. None of these.

18. A total-cost function is given by $C(x) = 1000\sqrt{x^3 + 1}$. Find the marginal cost when $x = 2$.
A. \$166.67 B. \$333.33 C. \$4000 D. \$2000 E. \$1000

19. A display case is in the shape of a rectangular box with a square base and open top. Suppose the volume is 21 cubic ft If x is the length of one side of the base, what value should x have to minimize the surface area? Give your answer correct to two decimal places.
A. 2.78 ft. B. 3.48 ft. C. 4.58 ft. D. 6.48 ft. E. 9.17 ft.

20. A manufacturer determines that in order to sell x units of a product, the price per unit must be $p = 1000 - x$. The manufacturer also determines that the total cost of producing x units is $C(x) = 3000 + 20x$. Calculate the maximum profit.
A. \$490 B. \$237,100 C. \$121,500 D. \$23,000 E. There is no maximum.

21. If $y = e^{x^2}$ then $\dfrac{dy}{dx} =$
A. e^{x^2} B. $x^2 e^{x^2-1}$ C. $2xe^{x^2-1}$ D. $2xe^{x^2}$ E. e^{2x}

22. If $y = \ln(1 - x^2)$ then $\dfrac{dy}{dx} =$
A. $\dfrac{1}{1-x^2}$ B. $\dfrac{2x}{\sqrt{1-x^2}}$ C. $\dfrac{-2x}{1-x^2}$ D. $\dfrac{1}{2(1-x^2)}$ E. $\dfrac{2x}{1-x^2}$

23. A population grows exponentially ($Q = Q_0 e^{kt}$). In 1960 it was 50,000 and in 1965 it was 100,000. What was the population in 1970?
A. 200,000 B. 150,000 C. 250,000 D. 300,000 E. 225,000

24. Evaluate $\int_0^4 3\sqrt{2x+1}dx$.
A. 27 B. 48 C. 52 D. 26 E. 35

25. Evaluate $\int (\frac{2}{x} - \sqrt{x})dx$.
A. $\ln x - 2/\sqrt{x} + C$ B. $-2/x^2 - x^{-1/2}/2 + C$ C. $2\ln x - 2x^{3/2}/3 + C$.
D. $-2/x^2 - 2x^{3/2}/3 + C$ E. $2\ln x - 1/2\sqrt{x} + C$

26. Evaluate $\int (3x-1)^{-4}dx$.
A. $(-12)(3x-1)^{-5} + C$ B. $-\frac{1}{9}(3x-1)^{-3} + C$ C. $(3x-1)^{-3} + C$
D. $-\frac{1}{3}(3x-1)^{-3} + C$ E. $-12(3x-1)^{-5} + C$

27. Evaluate $\int e^{3-2x}dx$.
A. $-2e^{3-2x} + C$ B. $-\frac{1}{2}e^{3-2x} + C$ C. $\frac{e^{4-2x}}{4-2x}$ D. $\frac{1}{3}e^{3-2x} + C$ E. $\frac{e^{3-2x}}{3-2x} + C$

28. Evaluate $\int_1^2 \frac{dx}{3x+1}$. Give your answer correct to four decimal places.
A. 0.5596 B. 0.6486 C. 1.9459 D. 0.0810 E. 0.1865

29. Evaluate $\int_0^1 x(x^2+1)^5 dx$.
A. $\frac{21}{4}$ B. $\frac{16}{3}$ C. $\frac{21}{2}$ D. $\frac{32}{3}$ E. $\frac{24}{3}$

30. The area of the region bounded by the curves $y = x^2 + 1$ and $y = 3x + 5$ is
A. $\frac{125}{6}$ B. $\frac{56}{3}$ C. $\frac{27}{2}$ D. $\frac{25}{6}$ E. $\frac{32}{3}$

31. Use the table of integrals to evaluate $\int 2x\ln x dx$.
A. $x^2\ln x - x^2/2 + C$ B. $\frac{1}{2}x^2\ln x - \frac{1}{2}x + C$ C. $\frac{1}{2}x^2\ln x - \frac{1}{6}x^3 + C$
D. $x\ln x^2 + 1/x + C$ E. None of these.

32. Use the table of integrals to evaluate $\int \frac{dx}{\sqrt{x^2-9}}$.
A. $\frac{1}{6}\ln|\frac{x-3}{x+3}| + C$ B. $\ln|x + \sqrt{x^2-9}| + C$ C. $\ln|x - \sqrt{x^2-9}| + C$ D. $\ln|x - \sqrt{x^2+9}| + C$
E. None of these.

33. Find the average value of $f(x) = x^2$ over the interval $1 \le x \le 4$.
A. $\frac{17}{2}$ B. $\frac{15}{2}$ C. 21 D. $\frac{65}{3}$ E. 7

34. Find a function f whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point (2,10). $f(x) =$
A. $-\frac{1}{3}(5-x^2)^{3/2}$ B. $\frac{2}{3}(5-x^2)^{3/2} + \frac{28}{3}$ C. $\frac{1}{3}(5-x^2)^{3/2} + \frac{29}{3}$ D. $-\frac{1}{3}(5-x^2)^{3/2} + \frac{31}{3}$
E. $\frac{3}{2}(5-x^2)^{3/2} + \frac{17}{2}$

35. It is estimated that t years from now the population of a certain town will be increasing at a rate of $5 + 3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will be population increase over the next 8 years?
A. 100 B. 9,760 C. 6,260 D. 24,760 E. 17,260

36. Calculate, if possible, the following improper integral $\int_0^\infty xe^{-x^2}dx$

    A. $-\frac{1}{2}$ B. 1 C. $\frac{1}{2}$ D. $\frac{5}{2}$ E. The integral diverges.

37. Find the value of k so that $f(x) = k(3-x)$ is a probability density function on the interval $[0, 3]$.

    A. $k = \frac{1}{9}$ B. $k = -\frac{2}{3}$ C. $k = -\frac{1}{3}$ D. $k = \frac{2}{9}$ E. $k = \frac{1}{6}$

38. Records indicate that t hours past midnight, the temperature at the West Lafayette airport was $f(t) = -0.3t^2 + 4t + 10$ degrees Celsius. What was the average temperature at the airport between 2:00 A.M. and 7:00 A.M.? (Give your answer to the nearest degree.)

    A. 3° B. 27° C. 21° D. 5° E. 18°

39. Let $f(x)$ be the probability density function defined on the interval $[0, \infty]$ by $f(x) = \dfrac{3}{x^4}$. Calculate $P(2 \leq x < \infty)$.

    A. 1 B. $\frac{3}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E.$\frac{1}{8}$

40. The probability density function for the life span of light bulbs manufactured by a certain company is $f(t) = 0.01e^{-0.01t}$ where t denotes the life span in hours of a randomly selected bulb, $0 \leq t < \infty$.. What is the probability that the life span of a randomly selected bulb is less than or equal to 10 hours? Give your answer correct to three decimal places.

    A. 0.009 B. 0.095 C. 0.905 D. 0.090 E. 0.303

Answers

1. B; 2. B; 3. C; 4. A; 5. B; 6. C; 7. D; 8. A; 9. A; 10. B; 11. A; 12. B; 13. D; 14. C; 15. B; 16. E; 17. D; 18. D; 19. B; 20. B; 21. D; 22. C; 23. A; 24. D; 25. C; 26. B; 27. B; 28. E; 29. A; 30. A; 31. A; 32. B; 33. E; 34. D; 35. B; 36. C; 37. D; 38. C; 39. E; 40. B