

The Formula Page may be used. It will be attached to the final exam.

- Calculate  $\frac{dy}{dx}$  if  $y = \cos(1 - 2x)$ .  
A.  $-\sin(1 - 2x)$  B.  $-2\sin(1 - 2x)$  C.  $2\sin(1 - 2x)$  D.  $\sin(1 - 2x)$  E.  $-2\cos(1 - 2x)$
- Find  $y'$  if  $y = x \tan^2 x$ .  
A.  $2x \tan x + \tan^2 x$  B.  $2x \tan x \sec^2 x$  C.  $x \sec^2 x + \tan^2 x$  D.  $2x \tan x \sec^2 x + \tan^2 x$   
E. None of these.
- If  $\sin \theta = -0.5473$  and  $\pi < \theta < \frac{3\pi}{2}$ , find  $\theta$  in radians. Give your answer correct to 4 decimal places.  
A.  $-0.5791$  B.  $3.7207$  C.  $3.1511$  D.  $2.5625$  E.  $1.761$
- Find the slope of the line perpendicular to the line containing the points  $(-2, 4)$  and  $(6, -3)$ .  
A.  $-8/7$  B.  $-7/8$  C.  $1/4$  D.  $8/7$  E.  $-1/2$
- Give the equation, in slope-intercept form, of the line which is parallel to the line  $2y - 6x - 5 = 0$  and passes through the point  $(-1, 3)$ .  
A.  $y = 3x + 10$  B.  $y = \frac{1}{3}x - 9$  C.  $y = \frac{1}{3}x + 2$  D.  $y = -3x$  E.  $y = 3x + 6$
- If  $f(x) = \frac{x}{x^2 + 1}$ , find  $\frac{1}{f(3)}$ .  
A.  $3/10$  B.  $3/16$  C.  $16/3$  D.  $10/3$  E.  $1/3$
- If  $f(x) = \frac{2}{x}$  then  $\frac{f(x + \Delta x) - f(x)}{\Delta x} =$   
A.  $\frac{-2}{x^2}$  B.  $\frac{2}{x + \Delta x} - \frac{2}{x}$  C.  $\frac{-2}{x(x + \Delta x)}$  D.  $\frac{2}{x(x + \Delta x)}$  E.  $\frac{-2}{x^2 + \Delta x}$
- $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 - 1} =$   
A. 3 B. 1 C.  $\infty$  D. 0 E. 5
- If the tangent line to the graph of  $y = f(x)$  at  $(2, 3)$  has equation  $x - y + 1 = 0$ , then  $f'(2) =$   
A. 1 B.  $3/2$  C.  $2/3$  D.  $-2/3$  E.  $-3/2$
- The derivative of  $\frac{x^2 + 1}{x + 5}$  is  
A.  $\frac{(x + 5)(2x) - (x^2 + 1)}{(x + 5)^2}$  B.  $2x$  C.  $\frac{(x + 5)(2x)}{(x^2 + 1)^2}$  D.  $\frac{(x^2 + 1) + (x + 5)(2x)}{(x + 5)^2}$  E.  $\frac{(x^2 + 1) - (x + 5)(2x)}{(x + 5)^2}$
- If  $y = (3 - x^2)^3$  then  $y'' =$   
A.  $-6x(3 - x^2)^2$  B.  $24x^2(3 - x^2) - 6(3 - x^2)^2$  C.  $6(3 - x^2)$  D.  $24x^2(3 - x^2)$  E. None of these.
- The line tangent to the graph of  $f(x) = x - \frac{1}{x}$  at  $(2, 3/2)$  has slope  
A.  $3/4$  B.  $3/2$  C. 0 D.  $1/4$  E.  $5/4$
- A point moves along the  $x$ -axis in such a way that its distance,  $s$ , from the origin at time  $t \geq 1$  is given by  $s = (t^3 - t)^{3/2}$ . Find the velocity of the point when  $t = 2$ . Give your answer correct to two decimal places.  
A. 3.67 B. 3.83 C. 40.42 D. 36.74 E. 21.08
- Differentiate  $y = \sin(x^2)$ .  $\frac{dy}{dx} =$   
A.  $\cos(x^2)$  B.  $2x \cos(x^2)$  C.  $-\cos(x^2)$  D.  $\cos(2x)$  E.  $-2x \cos(x^2)$

15. Find  $y'$  if  $y = x \tan x$ .  
 A.  $\sec^2 x$  B.  $x \sec^2 x$  C.  $x \tan 1 + \tan x$  D.  $1 + \sec^2 x$  E.  $x \sec^2 x + \tan x$
16. Find  $f'(x)$  if  $f(x) = \frac{x}{\cos(4x)}$ .  $f'(x) =$   
 A.  $\frac{\cos(4x) + 4x \sin(4x)}{\cos^2(4x)}$  B.  $\frac{-1}{4 \sin(4x)}$  C.  $\frac{\cos(4x) - x \sin(4x)}{\cos^2(4x)}$  D.  $\frac{\cos(4x) + x \sin(4x)}{\cos^2(4x)}$   
 E.  $\frac{-4x \sin(4x)}{\cos^2(4x)}$
17. The maximum value of  $f(x) = x^3(40 - x)^2$  on the closed interval,  $0 \leq x \leq 40$ , occurs at  $x =$   
 A. 20 B. 15 C. 35 D. 18 E. 24
18. Which of the following best describes the graph of  $y = 4x^3 - 3x^4$ ? The graph has a  
 A. relative maximum point and two points of inflection.  
 B. relative maximum point, a relative minimum point and one point of inflection.  
 C. a relative minimum point and two points of inflection.  
 D. a relative minimum point, two relative maximum points and two points of inflection.  
 E. None of these.
19. Which of the following best describes the graph of  $f(x) = \frac{x^2 + 1}{x^2 - 1}$ ?  
 A. Vertical asymptotes  $x = 1, x = -1$ , and symmetric to the  $x$ -axis.  
 B. Vertical asymptotes  $x = 1, x = -1$  but not symmetric to either the  $x$  or the  $y$ -axis.  
 C. Vertical asymptotes  $x = 1, x = -1$  and symmetric to the  $y$ -axis.  
 D. No vertical asymptotes and symmetric to the  $y$ -axis.  
 E. Vertical asymptote  $x = 1$  and symmetric to the  $x$ -axis.
20. Find the area of the region bounded by the curves  $x^2 + 4y = 0$  and  $x^2 - 4y - 8 = 0$ .  
 A.  $2/3$  B.  $16/3$  C. 6 D.  $4/3$  E.  $10/3$
21. What is the area of the largest rectangle with sides parallel to the axes which can be inscribed in the first quadrant under the parabola  $y = 4 - x^2$ ? (Give your answer correct to 2 decimal places.)  
 A. 1.15 B. 1.33 C. 3.08 D. 4.00 E. 2.67
22. A box with square base and no top is to have a volume of  $108 \text{ in.}^3$ . What is the smallest possible surface area of such a box.  
 A.  $32 \text{ in.}^2$  B.  $24 \text{ in.}^2$  C.  $256 \text{ in.}^2$  D.  $108 \text{ in.}^2$  E.  $56 \text{ in.}^2$
23. Find the value of  $dy$  and  $\Delta y$  for  $y = 2x^3 - 4x, x = 2$  and  $dx = \Delta x = 0.1$ . Give your answer correct to two decimal places.  
 A.  $dy = 2, \Delta y = 2.12$  B.  $dy = 2, \Delta y = 2.46$  C.  $dy = 2.12, \Delta y = 2$  D.  $dy = 2.46, \Delta y = 2$   
 E. None of these.
24. Calculate  $\lim_{x \rightarrow \infty} \frac{2 + 3x - 2x^3}{3 - 4x + x^3}$ .  
 A. 2 B.  $\infty$  C.  $2/3$  D.  $-3/4$  E.  $-2$
25. Evaluate  $\int \sqrt{2x + 1} dx$   
 A.  $\frac{2}{3}(2x + 1)^{3/2} + C$  B.  $\frac{1}{3}(2x + 1)^{3/2} + C$  C.  $(2x + 1)^{-1/2} + C$  D.  $2(2x + 1)^{1/2} + C$   
 E. None of these.

26. Evaluate  $\int_1^2 (6\sqrt{x} - \frac{1}{2\sqrt{x}})dx$ . Give your answer correct to 2 decimal places.  
 A. 9.90 B. 6.90 C. 5.66 D. 7.35 E. None of these
27. An object is thrown vertically downward from the top of a building 200 ft high with an initial velocity of 40 ft/sec. Find its velocity when it hits the ground. ( $s = -16t^2 - 40t + 200$ .)  
 A. -40 ft/sec B. -200 ft/sec C. -120 ft/sec D. -80 ft/sec E. None of these.
28. Calculate the area bounded by the parabola  $y = x^2$  and the line  $y = x + 2$ .  
 A. 9/2 B. 10/3 C. 7/6 D. 15/2 E. None of these.
29. Calculate the volume generated by revolving the area bounded by  $y = \sqrt{x}$ , the x-axis and  $x = 4$  about the y-axis. (Express your answer as a definite integral.)  
 A.  $\pi \int_0^4 x dx$  B.  $\pi \int_0^4 \sqrt{x} dx$  C.  $2\pi \int_0^4 (4-x)\sqrt{x} dx$  D.  $2\pi \int_0^4 x^2 dx$  E.  $2\pi \int_0^4 x^{3/2} dx$
30. Find the function,  $y$ , satisfying the following conditions:  $\frac{dy}{dx} = 3x^2 - 1$ , and the graph of  $y$  passes through the point (1, 3).  
 A.  $y = x^3 - x + 3$  B.  $y = x^3 - x$  C.  $y = 6x - 3$  D.  $y = 6x$  E.  $y = 3x^3 - x + 1$
31. Calculate the volume generated by revolving the area bounded by  $y = \sqrt{x}$ , the y-axis, and the line  $y = 2$ , about the x-axis. (Express your answer as a definite integral.)  
 A.  $\pi \int_0^4 (2 - \sqrt{x})^2 dx$  B.  $\pi \int_0^4 (4 - x) dx$  C.  $2\pi \int_0^4 (2 - \sqrt{x}) dx$  D.  $2\pi \int_0^4 (4\sqrt{x} - x) dx$   
 E.  $\pi \int_0^4 x dx$
32. If  $f'(x) = 4x - 3$  and  $f(0) = 7$  calculate  $f(2)$ .  
 A. 5 B. 2 C. 7 D. 9 E. 3
33. Calculate the centroid of a quarter circle of radius  $r$ .  
 A.  $\bar{x} = \frac{r}{3\pi}, \bar{y} = \frac{r}{3\pi}$  B.  $\bar{x} = \frac{4r}{3}, \bar{y} = \frac{4r}{3}$  C.  $\bar{x} = \frac{4r}{\pi}, \bar{y} = 0$  D.  $\bar{x} = \frac{4r}{\pi}, \bar{y} = \frac{4r}{\pi}$   
 E.  $\bar{x} = \frac{4r}{3\pi}, \bar{y} = \frac{4r}{3\pi}$
34. Calculate the  $x$ -coordinate of the centroid,  $\bar{x}$ , of the area given in problem 26, if the area is  $16/3$  square units.  
 A.  $\bar{x} = 2$  B.  $\bar{x} = 3/2$  C.  $\bar{x} = 9/5$  D.  $\bar{x} = 12/5$  E.  $\bar{x} = 1/5$
35. Find the work done in pumping the water out of the top of a cylindrical tank 5 ft in radius and 10 ft high, if the tank is initially half full of water, which weighs  $62.4 \text{ lb/ft}^3$ .  
 A.  $93,750\pi \text{ ft-lb}$  B.  $58,500\pi \text{ ft-lb}$  C.  $7,800\pi \text{ ft-lb}$  D.  $15,600\pi \text{ ft-lb}$  E. None of these.
36. A spring of natural length 12 ft. requires a force of 6 lb. to stretch it 2 ft. Find the work done in stretching it 6 ft. ( $F = kx$ )  
 A. 54 ft-lb B. 108 ft-lb C. 6 ft-lb D. 36 ft-lb E. 24 ft-lb
37. A vertical rectangular floodgate on a dam is 5 ft long and 4 ft deep. Find the force on the floodgate if its upper edge is 3 ft below the surface. (The weight of water is  $62.4 \text{ lb/ft}^3$ .) Give your answer correct to the nearest integer.  
 A. 7644 lb B. 3900 lb C. 1248 lb D. 6240 lb E. 2100 lb

38. A horizontal tank with vertical circular ends is filled with oil. If the radius of each end is 2 m, find the force on one end of the tank. (Assume  $w$  is the weight of the oil.) Express your answer as a definite integral. (Hint: Assume that the origin is at the center of one of the circular ends.)  
 A.  $2w \int_0^2 y\sqrt{4-y^2}dy$  B.  $w \int_{-2}^2 \sqrt{4-y^2}dy$  C.  $2w \int_{-2}^2 (2-y)\sqrt{4-y^2}dy$  D.  $2w \int_{-2}^2 (2-y)dy$   
 E. None of these.
39. Grant and Stadium Streets are straight and perpendicular to each other. A black 1997 Porsche 911 is going on Grant Street toward the intersection of the two streets at 60 mph (miles per hour), and a red 1993 Volkswagen Golf is going on Stadium Street toward the same intersection at a rate of 40 mph. At what rate is the distance between the two cars decreasing when the Porsche is  $1/2$  mile from the intersection and the Golf is  $3/8$  mile from it?  
 A. 40 mph B. 56 mph C. 60 mph D. 72 mph E. 32 mph
40. The line perpendicular to the graph of  $f(x) = x - \frac{1}{x}$  at  $(2, \frac{3}{2})$  has slope  
 A.  $3/4$  B.  $3/2$  C. 0 D.  $1/4$  E.  $-4/5$
41. Find the center  $C$  and radius  $r$  of the circle whose equation is  

$$x^2 + y^2 - 10x + 6y + 30 = 0$$
  
 A.  $C = (-5, 3); r = 2$  B.  $C = (-5, 3); r = 4$  C.  $C = (5, -3); r = 8$  D.  $C = (5, -3); r = 4$   
 E.  $C = (5, -3); r = 2$
42. If  $y^3 + x^2 = 9$  and  $\frac{dx}{dt} = 5$ , find  $\frac{dy}{dt}$  when  $x = 1$ .  
 A.  $-5/6$  B.  $2/3$  C.  $-10$  D.  $1/3$  E.  $10/3$
43. Water is flowing into a tank which is in the shape of a right circular cylinder standing on its circular base. If the water is flowing in at a rate of 80 cu. ft. per min. and the radius of the base of the tank is 4 ft., how fast is the water rising when the water is 10 ft. deep?  
 A.  $\frac{\pi}{5}$  ft/min B.  $5\pi$  ft/min C.  $\frac{50}{\pi}$  ft/min D.  $\frac{5}{\pi}$  ft/min E.  $50\pi$  ft/min
44. Find the mean value of  $f(x) = x^{1/3}$  on the interval  $[0, 2]$ .  
 A.  $\frac{3}{2^{2/3}}$  B.  $\frac{3}{4}$  C.  $\frac{1}{2^{4/3}}$  D.  $\frac{3}{2^{1/3}}$  E.  $\frac{3}{2^{5/3}}$
45. Find the root mean square of  $f(x) = \sqrt{x}(1-x)$  on the interval  $[0, 2]$ .  
 A.  $1/\sqrt{3}$  B.  $2/3$  C.  $2/\sqrt{3}$  D.  $\sqrt{3}/2$  E.  $1/\sqrt{6}$

### Answers

1. C; 2. D; 3. B; 4. D; 5. E; 6. D; 7. C; 8. A; 9. A; 10. A; 11. B; 12. E; 13. C; 14. B; 15. E; 16. A; 17. E; 18. A; 19. C; 20. B; 21. C; 22. D; 23. A; 24. E; 25. B; 26. B; 27. C; 28. A; 29. E; 30. A; 31. B; 32. D; 33. E; 34. D; 35. B; 36. A; 37. D; 38. C; 39. D; 40. E; 41. E; 42. A; 43. D; 44. E; 45. A.