

The Table of Integrals (Supplemental Appendix B of the text) and the Formula Page may be used. They will be attached to the final exam.

- If  $f(x, y) = (xy + 1)^2 - \sqrt{y^2 - x^2}$ , evaluate  $f(-2, 1)$ .  
A. 1 B.  $1 - \sqrt{5}$  C. Not defined D.  $-1 - \sqrt{5}$  E. None of these.
- A paint store carries two brands of latex paint. Sales figures indicate that if the first brand is sold for  $x_1$  dollars per gallon and the second for  $x_2$  dollars per gallon, the demand for the first brand will be  $D_1(x_1, x_2) = 100 + 5x_1 - 10x_2$  gallons per month and the demand for the second brand will be  $D_2(x_1, x_2) = 200 - 10x_1 + 15x_2$  gallons per month. Express the paint store's total monthly revenue,  $R$ , as a function of  $x_1$  and  $x_2$ .  
A.  $R = x_1D_1(x_1, x_2) + x_2D_2(x_1, x_2)$  B.  $R = D_1(x_1, x_2) + D_2(x_1, x_2)$   
C.  $R = D_1(x_1, x_2)D_2(x_1, x_2)$  D.  $R = x_2D(x_1, x_2) + x_1D_2(x_1, x_2)$  E. None of these.
- Compute  $\frac{\partial z}{\partial x}$ , where  $z = \ln(xy)$   
A.  $\frac{1}{x}$  B.  $\frac{1}{y}$  C.  $\frac{1}{xy}$  D.  $\frac{1}{x} + \frac{1}{y}$  E. None of these.
- Compute  $f_{uv}$  if  $f = uv + e^{u+2v}$   
A. 0 B.  $u + 2e^{u+2v}$  C.  $v + 2e^{u+2v}$  D.  $1 + 2e^{u+2v}$  E.  $1 + e^{u+2v}$
- If  $z = x^2 - xy$  and  $x = e^{2t}$  and  $y$  is a function of  $t$  then  $\frac{dz}{dt} =$   
A.  $(4xe^{2t} - 2ye^{2t})\frac{dy}{dt}$  B.  $-2xe^{2t}\frac{dy}{dt}$  C.  $4xe^{2t} - 2ye^{2t} - x\frac{dy}{dt}$  D.  $(2x - y)\frac{dy}{dt} - 2xe^{2t}$   
E. There is insufficient information to find  $\frac{dz}{dt}$ .
- Find and classify the critical points of  $f(x, y) = (x - 2)^2 + 2y^3 - 6y^2 - 18y + 7$   
A. (2,3) saddle point; (2,-1) relative minimum  
B. (2,3) relative maximum; (2,-1) relative minimum  
C. (2,3) relative minimum; (2,-1) relative maximum  
D. (2,3) relative maximum; (2,-1) saddle point  
E. None of these.
- A manufacturer sells two brands of foot powder, brand A and brand B. When the price of A is  $x$  cents per can and the price of B is  $y$  cents per can the manufacturer sells  $40 - 8x + 5y$  thousand cans of A and  $50 + 9x - 7y$  thousand cans of B. The cost to produce A is 10 cents per can and the cost to produce B is 20 cents per can. Determine the selling price of brand A which will maximize the profit.  
A. 40 cents B. 45 cents C. 15 cents D. 50 cents E. None of these.
- Use the total differential to estimate the change in  $z$  at (1,3) if  $\frac{\partial z}{\partial x} = 2x - 4$ ,  $\frac{\partial z}{\partial y} = 2y + 7$ , the change in  $x$  is 0.3 and the change in  $y$  is 0.5.  
A. 7.1 B. 2.9 C. 4.9 D. 5.9 E. None of these.

9. Using  $x$  worker-hours of skilled labor and  $y$  worker-hours of unskilled labor, a manufacturer can produce  $f(x, y) = x^2y$  units. Currently 16 worker-hours of skilled labor and 32 worker-hours of unskilled labor are used. If the manufacturer increases the unskilled labor by 10 worker-hours, use calculus to estimate the corresponding change that the manufacturer should make in the level of skilled labor so that the total output will remain the same.  
 A. Reduce by 4 hours. B. Reduce by 10 hours. C. Reduce by  $\frac{5}{4}$  hours.  
 D. Reduce by  $\frac{5}{2}$  hours. E. None of these.
10. Find the maximum value of the function  $f(x, y) = 20x^{3/2}y$  subject to the constraint  $x + y = 60$ . (Give your answer to the nearest integer.)  
 A. 84,654 B. 188,334 C. 4,320 D. 259,200 E. 103,680
11. Evaluate  $\int (\frac{2}{x} - \sqrt{x}) dx$ .  
 A.  $\ln|x| - 2/\sqrt{x} + C$  B.  $-2/x^2 - x^{-1/2}/2 + C$  C.  $2 \ln|x| - 2x^{3/2}/3 + C$ .  
 D.  $-2/x^2 - 2x^{3/2}/3 + C$  E. None of these.
12. Evaluate  $\int (3x - 1)^{-4} dx$ .  
 A.  $(-12)(3x - 1)^{-5} + C$  B.  $-\frac{1}{9}(3x - 1)^{-3} + C$  C.  $(3x - 1)^{-3} + C$   
 D.  $-\frac{1}{3}(3x - 1)^{-3} + C$  E. None of these.
13. Evaluate  $\int e^{3-2x} dx$ .  
 A.  $-2e^{3-2x} + C$  B.  $-\frac{1}{2}e^{3-2x} + C$  C.  $\frac{e^{4-2x}}{4-2x}$  D.  $\frac{1}{3}e^{3-2x} + C$  E. None of these
14. A manufacturer has found that the marginal cost is  $6q + 1$  dollars per unit when  $q$  units have been produced. The total cost of producing the first unit is \$130. What is the total cost of producing the first 10 units?  
 A. \$436 B. \$566 C. \$310 D. \$440 E. None of these.
15. Find a function  $f$  whose tangent line has slope  $x\sqrt{5-x^2}$  for each value of  $x$  and whose graph passes through the point  $(2, 10)$ .  $f(x) =$   
 A.  $-\frac{1}{3}(5-x^2)^{3/2}$  B.  $\frac{2}{3}(5-x^2)^{3/2} + \frac{28}{3}$  C.  $\frac{1}{3}(5-x^2)^{3/2} + \frac{29}{3}$  D.  $-\frac{1}{3}(5-x^2)^{3/2} + \frac{31}{3}$   
 E. None of these.
16. Use integration by parts to evaluate  $\int x \ln(x^2) dx$ .  
 A.  $x^2 \ln x - x^2/2 + C$  B.  $\frac{1}{2}x^2 \ln x^2 - \frac{1}{2}x + C$  C.  $\frac{1}{2}x^2 \ln x^2 - \frac{1}{6}x^3 + C$   
 D.  $x \ln x^2 + 1/x + C$  E. None of these.
17. The area of the region bounded by the curves  $y = x^2 + 1$  and  $y = 3x + 5$  is  
 A.  $\frac{125}{6}$  B.  $\frac{56}{3}$  C.  $\frac{27}{2}$  D.  $\frac{25}{6}$  E. None of these.
18. Find the average value of  $f(x) = x^2$  over the interval  $1 \leq x \leq 4$ .  
 A.  $\frac{17}{2}$  B.  $\frac{15}{2}$  C. 21 D.  $\frac{65}{3}$  E. 7
19. A calculator manufacturer expects that  $x$  months from now consumers will be buying 1000 calculators a month at a price of  $20 + 3\sqrt{x}$  dollars per calculator. What is the total revenue the manufacturer can expect from the sale of calculators over the next 4 months?  
 A. \$8,000 B. \$16,000 C. \$96,000 D. \$192,000 E. None of these.

20. The general solution of the differential equation  $\frac{dy}{dx} = 2y + 1$  is  
 A.  $x = y^2 + y + C$  B.  $2y + 1 = Ce^{2x}$  C.  $y = 2xy + x + C$  D.  $y = Ce^{2x} - 2y - 1$   
 E. None of these.
21. The value,  $V$ , of a certain \$1500 IRA account grows at a rate equal to 13.5% of its value. Its value after  $t$  years is  
 A.  $V = 1500e^{-0.135t}$  B.  $V = 1500 + 0.135t$  C.  $V = 1500e^{0.135t}$  D.  $V = 1500(1 + 0.135t)$   
 E.  $V = 1500 \ln(0.135t)$
22. A tank currently holds 200 gallons of brine that contains 4 pounds of salt per gallon. Brine containing 2 pounds of salt per gallon flows into the tank at the rate of 5 gallons per minute, while the mixture, which is kept uniform, runs out of the tank at the same rate. Express the amount of salt  $Q$  in the tank as a function of time  $t$ .  
 A.  $Q(t) = 400 + 200e^{-t/40}$  B.  $Q(t) = 800 + 400e^{-t/40}$  C.  $Q(t) = 800e^{-t/40}$   
 D.  $Q(t) = 400 + 400e^{-t/40}$  E. None of these.
23. Evaluate  $\int \frac{dx}{\sqrt{9x^2 - 4}}$ , using the Table of Integrals in the text.  
 A.  $\ln|3x + \sqrt{9x^2 - 4}| + C$  B.  $\frac{1}{3} \ln|x + \sqrt{x^2 - 4}| + C$  C.  $\frac{1}{3} \ln|x + \sqrt{x^2 + (4/9)}| + C$   
 D.  $\frac{1}{3} \ln|x + \sqrt{x^2 - (4/9)}| + C$  E.  $\ln|x + \sqrt{x^2 - (4/9)}| + C$ .
24. Use integration by parts to evaluate  $\int_1^2 x \ln(x^2) dx$ . (Give your answer correct to two decimal places.)  
 A. 1.27 B. -0.30 C. 2.27 D. 0.77 E. None of these.
25. It is estimated that  $t$  years from now the population of a certain town will be increasing at a rate of  $5 + 3t^{2/3}$  hundred people per year. If the population is presently 100,000, by how many people will the population increase over the next 8 years?  
 A. 100 B. 9,760 C. 6,260 D. 109,760 E. None of these.
26. The probability density function for the life span of light bulbs manufactured by a certain company is  $f(x) = 0.01e^{-0.01x}$  where  $x$  denotes the life span in hours of a randomly selected bulb. What is the probability that the life span of a randomly selected bulb is less than or equal to 10 hours? Give your answer correct to three decimal places.  
 A. 0.009 B. 0.095 C. 0.905 D. 0.090 E. None of these.
27. Calculate, if possible, the following improper integral  $\int_0^{\infty} xe^{-x^2} dx$   
 A.  $-\frac{1}{2}$  B. 1 C.  $\frac{1}{2}$  D. The integral diverges. E. None of these.
28. An object moves so that its velocity after  $t$  minutes is given by the formula  $v = 20e^{-0.01t}$ . The distance it travels during the 10th minute is  
 A.  $\int_0^{10} 20e^{-0.01t} dt$  B.  $\int_9^{10} (-20e^{-0.01t}) dt$  C.  $\int_0^{10} (-20e^{-0.01t}) dt$   
 D.  $\int_9^{10} 20e^{-0.01t} dt$  E. None of these.

29. A certain traffic light remains red for 50 seconds at a time. You arrive (at random) at the light and find it red. What is the probability that you will have to wait more than 20 seconds for the light to turn green?  
 A.  $\frac{3}{5}$  B.  $\frac{2}{5}$  C.  $\frac{4}{5}$  D.  $\frac{1}{5}$  E. None of these.
30. Find the value of  $k$  so that  $f(x) = k(3 - x)$  for  $0 \leq x \leq 3$  is a probability density function. (Assume  $f(x) = 0$  if  $x$  is not in the interval  $0 \leq x \leq 3$ .)  
 A.  $k = \frac{1}{9}$  B.  $k = -\frac{2}{3}$  C.  $k = -\frac{1}{3}$  D.  $k = \frac{2}{9}$  E. None of these.
31. What is the expected value of the random variable given in problem 26?  
 A. 10 hrs. B. 100 hrs. C. 50 hrs. D. 95 hrs. E. None of these.
32. Records indicate that  $t$  hours past midnight, the temperature at the West Lafayette airport was  $f(t) = -0.3t^2 + 4t + 10$  degrees Celsius. What was the average temperature at the airport between 2:00 A.M. and 7:00 A.M.? (Give your answer to the nearest degree.)  
 A.  $3^\circ$  B.  $27^\circ$  C.  $21^\circ$  D.  $5^\circ$  E. None of these.
33. Suppose  $f(x)$  is the probability density function of a random variable  $X$ , where

$$f(x) = \begin{cases} \frac{3}{x^4} & \text{if } 1 \leq x < \infty, \\ 0 & \text{if } x < 1. \end{cases}$$

Calculate  $P(2 \leq X < \infty)$ .

- A. 1 B.  $\frac{3}{8}$  C.  $\frac{1}{4}$  D.  $\frac{1}{2}$  E.  $\frac{1}{8}$
34. Approximate  $\int_0^1 e^{x^2} dx$  using the trapezoidal rule with  $n = 4$ . (Give the answer correct to two decimal places.)  
 A. 1.49 B. 2.98 C. 5.96 D. 1.73 E. None of these.
35. Determine the smallest number of subintervals required to guarantee accuracy to within 0.005 in the approximation of  $\int_0^1 e^{x^2} dx$  using the trapezoidal rule.  
 A. 10 B. 15 C. 17 D. 21 E. None of these.
36.  $\lim_{x \rightarrow \infty} \frac{x^2 + 4x - 5}{3x^2 - 1} =$   
 A. 3 B. 0 C.  $\infty$  D.  $\frac{1}{3}$  E. None of these.
37. Calculate  $\lim_{x \rightarrow \infty} \frac{\ln(2x + 1)}{\ln(3x + 1)}$ .  
 A.  $\frac{2}{3}$  B.  $\frac{3}{2}$  C. 0 D.  $\infty$  E. 1
38. The slope of the least-squares line for the points (1,2), (2,4), (4,4), (5,2) is  
 A. 0 B. 1 C. 2 D. 3 E. None of these.
39. Find the sum of the series
- $$\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n.$$
- A.  $\frac{2}{5}$  B.  $-\frac{2}{5}$  C.  $\frac{3}{2}$  D.  $-\frac{3}{2}$  E. The series diverges.

40. Money is transferred continuously into an account at the constant rate of \$5,000 per year and the account earns interest at the annual rate of 7 percent compounded continuously. How much will be in the account at the end of 5 years.  
A. \$34,219 B. \$28,697 C. \$32,417 D. \$29,933 E. \$28,145
41. Use a Taylor polynomial of degree 2 to approximate the number  $\sqrt{3.8}$  rounded to five decimal places.  
A. 1.94936 B. 1.94938 C. 1.94940 D. 1.94947 E. 1.95000
42. Use a Taylor polynomial of degree 2 to approximate the integral  $\int_0^{0.1} \frac{100}{x^2+1} dx$  rounded to five decimal places.  
A. 9.96687 B. 10.00000 C. 9.96677 D. 9.66667 E. 9.96667

### Answers

1. C; 2. A; 3. A; 4. D; 5. C; 6. E ((2,3) is a relative minimum, (2, -1) is a saddle point); 7. A; 8. D; 9. D; 10. E; 11. C; 12. B; 13. B; 14. A; 15. D; 16. A; 17. A; 18. E; 19. C; 20. B; 21. C; 22. D; 23. D; 24. A; 25. B; 26. B; 27. C; 28. D; 29. A; 30. D; 31. B; 32. C; 33. E; 34. A; 35. C; 36. D; 37. E; 38. A; 39. B; 40. D; 41. B; 42. E