

MA 224 FORMULAS

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y , and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f .

1. If $D(a, b) < 0$, then f has a saddle point at (a, b) ,
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b) .
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b) .
4. If $D(a, b) = 0$, the test is inconclusive.

LEAST-SQUARES LINE

The equation of the least-squares line for the n points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, is $y = mx + b$, where

$$m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}, \quad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}.$$

TRAPEZOIDAL RULE

$$\int_a^b f(x) dx \equiv \frac{\Delta x}{2} \left[f(x_1) + 2f(x_2) + \cdots + 2f(x_n) + f(x_{n+1}) \right],$$

$$\text{where } x_{i+1} - x_i = \Delta x = \frac{b-a}{n}, \quad x_1 = a, \quad x_{n+1} = b.$$

ERROR ESTIMATE FOR THE TRAPEZOIDAL RULE

If M is the maximum value of $|f''(x)|$ on the interval $a \leq x \leq b$, then

$$|E_n| \leq \frac{M(b-a)^3}{12n^2}$$

PROBABILITY

If $f(x)$ is a probability density function, then

$$\text{Expected Value (Mean)} = E = \int_{-\infty}^{\infty} xf(x) dx$$

$$\text{Variance} = V = \int_{-\infty}^{\infty} x^2 f(x) dx - E^2$$

GEOMETRIC SERIES

If $|r| < 1$, with $r \neq 0$, then

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}.$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = c$ is the power series $\sum_{n=0}^{\infty} a_n(x-c)^n$, where

$$a_n = \frac{f^{(n)}(c)}{n!}.$$

Examples: (with $c = 0$)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \text{for } -\infty < x < \infty; \quad \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n, \quad \text{for } -1 < x \leq 1.$$