MA 262 Spring 2003
FINAL EXAM INSTRUCTIONS

NAME ______________________ INSTRUCTOR ______________________

**INSTRUCTIONS:**

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the instructor's name and the course number.
3. Fill in your name and student identification number and blacken in the appropriate spaces.
4. Mark in the section number, the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)
5. Sign the mark–sense sheet.
6. Fill in your name and your instructor's name above.
7. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.
8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.
9. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

NAME ______________________ INSTRUCTOR ______________________

1. Let $y$ be the solution of $y' = \frac{y}{x} + xe^x$ that satisfies $y(1) = 0$. Then $y(2) =$

A) $2e^2 - 2e$

B) $e^2 - e$

C) $2e^2 - e$

D) $2e^2$

E) $0$

2. A tank initially contains $100L$ of a solution in which is dissolved $50g$ of chemical. Water flows into the tank at the rate of $4L/\text{min}$, and the well-mixed solution flows out at the same rate. What is the amount of chemical (in grams) in the tank after 50 minutes?

A) $25e^{-4}$

B) $5e^{-4}$

C) $50e^{-2}$

D) $25e^{-2}$

E) $100-50e^{-2}$

3. The general solution of $(2xy^2 + 2x) + (2x^2y - \sin y)y' = 0$ is given by:

A) $x^2y^2 + 2xy + \cos y = C$

B) $\frac{2}{3}xy^3 + 2xy = C$

C) $x^2y^2 + x^2 = C$

D) $x^2y^2 + x^2 + \cos y = C$

E) $\frac{2}{3}x^3y - x\sin y = C$

4. One solution of the equation $xy'' + (1 - 2x)y' + (x - 1)y = 0$ is $y_1 = e^x$. If another solution has the form $u(x) \cdot e^x$ then $u$ must satisfy:

A) $xu'' + u' = 0$

B) $u'' + u' = 0$

C) $u'' + xu' = 0$

D) $xu'' + e^x u' = 0$

E) $e^x u'' + xu' = 0$

5. A particular solution of $y'' - 2y' + y = \sin x + e^x$ (for a suitable choice of constants) is:

A) $A\sin x + B + \cos x + Ce^x$

B) $A\sin x + B\cos x + Ce^x + Dxe^x$

C) $A\sin x + Bx^2e^x$

D) $A\sin x + B\cos x + Cxe^x$

E) $A\sin x + B\cos x + Cx^2e^x$

6. A basis of solutions to $(D^2 + 4D + 4)y = 0$ is:

A) $\cos 2x, \sin 2x$

B) $e^{2x}, xe^{2x}$

C) $e^{-2x}, xe^{-2x}$

D) $e^{2x}, xe^{2x}, x^2e^{2x}$

E) $e^{-2x}, xe^{-2x}, x^2e^{-2x}$

7. If $y = u_1 y_1 + u_2 y_2$ is a particular solution of the equation $y'' - 2y' - 3y = 4e^{4x}$, where $y_1 = e^{-x}$ and $y_2 = e^{3x}$, then $u_1 - u_2$ is

A) $-e^{x} - \frac{e^{5x}}{5}$

B) $e^{x} + \frac{e^{5x}}{5}$

C) $e^{4x} + e^{x}$

D) $\frac{e^{4x}}{4} - 4e^{7x}$

E) $\frac{e^{4x}}{4} - e^{x}$

8. Let $A = \begin{bmatrix} 2 & 1 & 3 & 4 & 2 \\ 1 & 0 & 2 & 1 & 3 \\ 2 & 3 & 1 & 5 & 7 \end{bmatrix}$. The rank of $A$ is:

A) 1

B) 2

C) 3

D) 4

E) 5

9. The system

$$x + y - z = 3$$
$$x - y + 3x = 4$$
$$x + y + (k^2 - 10)z = k$$

has infinitely many solutions if

A) $k = -3$

B) $k = 4$

C) $k = 0$

D) $k = 3$

E) $k = 1$

10. Let $B = \begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \\ -3 & 0 & 8 \end{bmatrix}$ and let $C$ be a $3 \times 3$ matrix with $\det C = 6$. If $P$ is a nonsingular $3 \times 3$ matrix, then $\det(3BP^{-1}C^T P)$ is:

A) $-324$

B) $-108$

C) $-36$

D) $-2$

E) $0$

11. Let $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$. Then $\det A$ is:

A) 6

B) 1

C) 0

D) $-2$

E) $-4$

12. Let $E = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$. The element in the first row and third column of $E^{-1}$ is:

A) 1

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $-\frac{1}{2}$

E) $-1$

13. A basis for the subspace of $R^4$ spanned by the vectors $\{(1,2,-1,0),(4,8,-4,-3),(0,1,3,4),(2,5,1,4)\}$ is:

A) $\begin{bmatrix}1\\0\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$

B) $\begin{bmatrix}1\\2\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-3\\4\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

C) $\begin{bmatrix}1\\0\\-7\\0\end{bmatrix}, \begin{bmatrix}0\\1\\3\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

D) $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

E) $\begin{bmatrix}1\\1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$

14. Let $W \subset R^4$ be the subspace of solutions of the system of linear equations $AX = 0$, where $A = \begin{bmatrix}2 & 1 & 2 & 3\\1 & 1 & 3 & 0\end{bmatrix}$. Find a basis for $W$.

A) $\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-3\\0\\3\\1\end{bmatrix}$

B) $\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

C) $\begin{bmatrix}1\\0\\-1\\3\end{bmatrix}, \begin{bmatrix}0\\1\\4\\-3\end{bmatrix}$

D) $\begin{bmatrix}1\\-4\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\3\\0\\1\end{bmatrix}$

E) $\begin{bmatrix}1\\5\\-2\\-1\end{bmatrix}, \begin{bmatrix}2\\-1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-4\\1\\0\end{bmatrix}$

15. One eigenvalue of $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$ is $\lambda = 2$. A basis for the corresponding eigenspace is:

A) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

D) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

E) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

16. For what value(s) of $k$ will the functions $2x - 1$, $kx^2 - 1$, and $3x - k + 1$ be linearly independent over $R$?

A) $k \neq 0, k \neq \frac{-5}{2}$

B) $k = 0, k = \frac{5}{2}$

C) $k = 0, k = \frac{-5}{2}$

D) $k = 0$

E) $k \neq 0, k \neq \frac{5}{2}$

17. Which of the following sets $S$ are subspaces of the given vector space $V$?

(1) $S = \{(x, y, z) \in V | x + y + 2z = 0\}$, where $V = R^3$

(2) $S = \{y \in V | y'' + 2y' + y = 1\}$, where $V = C^2(I)$

(3) $S = \{A \in V | A = A^T\}$, where $V = M_3(R) =$ the vector space of $3 \times 3$ matrices with real elements

(4) $S = \{A \in V | \det(A) = 1\}$, where $V = M_2(R) =$ the vector space of $2 \times 2$ matrices of real elements

A) (3) only

B) (1) and (3)

C) (1) and (2)

D) (2) and (4)

E) (3) and (4)

18. Which of the following matrices are nondefective?

$A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$?

A) $A$ only

B) $B$ only

C) $A$ and $B$

D) $A$ and $C$

E) $B$ and $C$

19. Let $T : R^2 \to R^2$ be the linear transformation such that $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, and $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then $T\left(2\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ is

A. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

20. The general solution of $y''' - 3y'' + 4y' - 2y = 0$ is

A) $y = c_1e^x + c_2 \cos x + c_3 \sin x$

B) $y = c_1e^x + c_2e^x \cos x + c_3e^x \sin x$

C) $y = c_1e^x + c_2e^2x + c_3$

D) $y = c_1e^x + c_2e^{(1+\sqrt{3})x} + c_3e^{(1-\sqrt{3})}x$

E) $y = c_1e^x + c_2xe^x + c_3x^2e^x$

21. Which form should you use to determine a particular solution to the equation

$$y''' + 8y = t^3 + 2e^{-2t}$$

using the method of undetermined coefficients?

A) $y_p(t) = At^3 + Bt^2 + Ct + D + Ee^{-2t}$

B) $y_p(t) = At^3 + Be^{-2t}$

C) $y_p(t) = At^3 + Bt^3e^{-2t}$

D) $y_p(t) = At^3 + Bt^2 + Ct + D + Ete^{-2t}$

E) $y_p(t) = At^3 + Bt^3e^{-2t} + Ct^2e^{-2t} + Dte^{-2t} + Ee^{-2t}$

22. The system

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 8e^t \\ -4e^t \end{bmatrix}$$

has fundamental matrix

$$X(t) = \begin{bmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{bmatrix}.$$

A particular solution $\vec{x}_p(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ has $x_2(t)$ equal to:

A) $-8e^t$

B) $-2e^t$

C) $-8e^{-t}$

D) $-2e^{-t}$

E) $8e^{-t}$

23. An annihilator of $x^3e^x$ is:

A) $(D-1)^3$
B) $(D-1)^4$
C) $(D+1)^3$
D) $(D+1)^4$
E) $D^3(D^2+2D+1)$

24. Three independent solutions to

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

are

A) $\begin{bmatrix} 3e^t \\ e^t \\ -e^t \end{bmatrix}, \begin{bmatrix} 0 \\ -2e^{-t} \\ e^{-t} \end{bmatrix}, \begin{bmatrix} 0 \\ e^{4t} \\ 2e^{4t} \end{bmatrix}$

B) $\begin{bmatrix} e^t \\ e^t \\ 0 \end{bmatrix}, \begin{bmatrix} e^{-t} \\ 0 \\ e^{-t} \end{bmatrix}, \begin{bmatrix} 0 \\ e^{2t} \\ e^{2t} \end{bmatrix}$

C) $\begin{bmatrix} e^t \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} e^{-t} \\ 0 \\ e^{-t} \end{bmatrix}, \begin{bmatrix} 0 \\ e^{4t} \\ 2e^{4t} \end{bmatrix}$

D) $\begin{bmatrix} e^t \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2e^{-t} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ e^{4t} \\ -e^{4t} \end{bmatrix}$

E) $\begin{bmatrix} e^t \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2e^{-t} \\ e^{-t} \end{bmatrix}, \begin{bmatrix} 0 \\ e^{2t} \\ 2e^{2t} \end{bmatrix}$

25. Two independent solutions to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

are

A) $\begin{bmatrix} \cos t \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sin t \end{bmatrix}$

B) $\begin{bmatrix} e^t \cos t \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ e^t \sin t \end{bmatrix}$

C) $\begin{bmatrix} -2\cos t - \sin t \\ 5\cos t \end{bmatrix}, \begin{bmatrix} -2\sin t + \cos t \\ 5\sin t \end{bmatrix}$

D) $\begin{bmatrix} e^t \\ e^{-t} \end{bmatrix}, \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

E) $\begin{bmatrix} e^t \\ e^t \end{bmatrix}, \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$