

Linear Systems

Consider

$$X' = AX$$

where A is nxn .If there are n linearly independent eigenvectors for A , $\{v(1), \dots, v(n)\}$, then a fundamental set of solutions is given by $\{\exp(s(i)*t)v(i), i=1..n\}$. If n is large though this may be hard to find by hand. Here we show how to use Matlab to find the eigenvalues and eigenvectors.

example

Let

A =

$$\begin{bmatrix} -1 & 0 & 2 \\ 2 & 3 & 6 \\ -2 & 0 & -1 \end{bmatrix}$$

Type :

[E,V]=eig(A)

We get

E =

$$\begin{bmatrix} 0 & 0.5000 & 0.5000 \\ 1.0000 & 0.1000- 0.7000i & 0.1000+ 0.7000i \\ 0 & 0+ 0.5000i & 0- 0.5000i \end{bmatrix}$$

V =

$$\begin{bmatrix} 3.0000 & 0 & 0 \\ 0 & -1.0000+ 2.0000i & 0 \\ 0 & 0 & -1.0000- 2.0000i \end{bmatrix}$$

The diagonal matrix V lists the eigenvalues of A and the corresponding columns of E are the eigenvectors. Using this information we can write down 3 linearly independent real solutions (a fundamental set).

$$X(1) = \exp(3*t) * [0, 1, 0]'$$

$$X(2) = \exp(-t) * [0.5*\cos(2*t), 0.1*\cos(2*t) + \sin(2*t)*0.7, -0.5*\sin(2*t)]'$$

$$X(3) = \exp(-t) * [0.5*\sin(2*t), -0.7*\cos(2*t) + 0.1*\sin(2*t), 0.5*\cos(2*t)]'$$

Note v' is the transpose of v .

What do you do if there are not n linearly independent eigenvectors? To see how to complete a fundamental set in this case look at Polking, pages 163-170..

ASSIGNMENT 5 :

Write down a real fundamental set if

1.

A =

-0.2	0	0.2
0.2	-0.4	0
0	0.4	-0.2

2.

A =

4	1	1	7
1	4	10	1
1	10	4	1
7	1	1	4