MATLAB.8

Partial Differential Equations
Solving the Heat Equation .

Let f(x)=0 if 0<x<1,
=1 if 1<x<2.

Consider the problem

d^2u/dxdx=du/dt 0<x<2, t>0
u(0,t)=u(2,t)=0 t>0,
u(x,0)=f(x) .

The Fourier sine coefficients for f are
b(m)=(cos(m*pi/2)-cos(m*pi))*2/m/pi

The M-file computing the first n terms is

```
**********************************************************************
function w=U(x,t,n)
w=0;
for m=1:n
 w=w+2*(cos(m*pi/2)-cos(m*pi))/m/pi*exp(-m^2*pi^2*t/4).*sin(m*pi*x/2);
end
**********************************************************************
```

Notice the .* above. We will do a 3-dimensional plot and U.m needs to be array smart for this.
Let's set n=50.
To plot the temperature distribution at different times enter

```
**********************************************************************
hold on
fplot('U(x,0,50)',[0,2])
fplot('U(x,.1,50)',[0,2])
fplot('U(x,.5,50)',[0,2])
fplot('U(x,1,50)',[0,2])
fplot('U(x,2,50)',[0,2])
fplot('U(x,5,50)',[0,2])
hold off
**********************************************************************
```

To do a 3-D plot on the x-t rectangle 0<x<2,0<t<1 you subdivide it into rectangles of side length dx by dt and plot the u at the verticies.
Here we choose dx=.05 and dt=.05.
Enter

```
************************************************
x=0:.05:2;
t=0:.05:1;
[x,t]=meshgrid(x,t);
mesh(x,t,U(x,t,50))
************************************************
```

ASSIGNMENT 8 :

1. HEAT EQUATION

Let u model the temperature in a rod and solve

d^2u/dxdx=du/dt 0<x<40, t>0

u(0,t)=u(40,t)=0 t>0,

u(x,0)=x for 0<x<40 .

a) Find the series solution and consider the partial sum U=U(x,t,50).

b) Plot U versus x for t=5,10,20,40,100 and 200.

c) For each value of t used in b) estimate the value of x for which the value of U is greatest . (See HW.2 .) Plot these values versus t to see how the warmest point in the rod changes with time.

d) Plot U versus t for x=10,20 and 30. The command fplot only recognizes "x" as an independent variable. For example if you have an M-file for a function g(x,t), and you want the graph of g(3,t) for 1<t<3 type :
fplot('g(3,x)',[1,3]) .

e) Do a 3-D plot of U versus x and t.

f) How long does it take for the entire rod to cool off to a temperature of no more than 2 degrees ?

2. WAVE EQUATION

Let u model the displacement of a vibrating string and solve

d^2u/dxdx=d^2u/dtdt 0<x<10, t>0

u(0,t)=u(10,t)=0 t>0,

u(x,0)=1 for 4<x<6
=0 for 0<x<4 & 6<x<10

du/dt(x,0)=0 for 0<x<10 .

a) Find the series solution and consider the partial sum U=U(x,t,50).

b) Plot U versus x for t=0,2,4,...18,20 .Each on its own axes.

c) Plot U versus t for 0<t<20 for x= 5 and x=2.5 on separate axes.

d) Describe the motion of the string in a few sentences.