

Linear Systems

Consider

$$X' = AX$$

where A is $n \times n$. If there are n linearly independent eigenvectors for A , $\{v(1), \dots, v(n)\}$, then a fundamental set of solutions is given by $\{\exp(s(i)*t)v(i), i=1..n\}$. If n is large though this may be hard to find by hand. Here we show how to use Matlab to find the eigenvalues and eigenvectors.

example

Let
A =

$$\begin{bmatrix} -1 & 0 & 2 \\ 2 & 3 & 6 \\ -2 & 0 & -1 \end{bmatrix}$$

Type :

```
*****  
[E,V]=eig(A)  
*****
```

We get

E =

$$\begin{bmatrix} 0 & 0.5000 & 0.5000 \\ 1.0000 & 0.1000 - 0.7000i & 0.1000 + 0.7000i \\ 0 & 0 + 0.5000i & 0 - 0.5000i \end{bmatrix}$$

V =

$$\begin{bmatrix} 3.0000 & 0 & 0 \\ 0 & -1.0000 + 2.0000i & 0 \\ 0 & 0 & -1.0000 - 2.0000i \end{bmatrix}$$

The diagonal matrix V lists the eigenvalues of A and the corresponding columns of E are the eigenvectors. Using this information we can write down 3 linearly independent real solutions (a fundamental set).

```
X(1)=exp(3*t)*[0,1,0]'  
X(2)=exp(-t)*[0.5*cos(2*t),0.1*cos(2*t)+sin(2*t)*0.7,-0.5*sin(2*t)]'  
X(3)=exp(-t)*[0.5*sin(2*t),-0.7*cos(2*t)+0.1*sin(2*t),0.5*cos(2*t)]'  
Note v' is the transpose of v .
```

What do you do if there are not n linearly independent eigenvectors? To see how to complete a fundamental set in this case look at Polking, pages 163-170..

ASSIGNMENT 4 :

Write down a real fundamental set if

1.

A =

-0.2	0	0.2
0.2	-0.4	0
0	0.4	-0.2

2.

A =

4	1	1	7
1	4	10	1
1	10	4	1
7	1	1	4