

Partial Differential Equations Solving the Heat Equation .

Let $f(x)=0$ if $0<x<1$,
 $=1$ if $1<x<2$.

Consider the problem

$$\begin{aligned} d^2u/dx^2 &= du/dt & 0 < x < 2, \quad t > 0 \\ u(0,t) &= u(2,t) = 0 & t > 0, \\ u(x,0) &= f(x) \end{aligned}$$

The Fourier sine coefficients for f are

$$b(m) = (\cos(m\pi/2) - \cos(m\pi)) * 2/m\pi$$

The M-file computing the first n terms is

```
*****
function w=U(x,t,n)
w=0;
for m=1:n
    w=w+2*(cos(m*pi/2)-cos(m*pi))/m/pi*exp(-m^2*pi^2*t/4).*sin(m*pi*x/2);
end
*****
```

Notice the `.*` above. We will do a 3-dimensional plot and `U.m` needs to be array smart for this.

Let's set $n=50$.

To plot the temperature distribution at different times enter

```
*****
hold on
fplot('U(x,0,50)', [0,2])
fplot('U(x,.1,50)', [0,2])
fplot('U(x,.5,50)', [0,2])
fplot('U(x,1,50)', [0,2])
fplot('U(x,2,50)', [0,2])
fplot('U(x,5,50)', [0,2])
hold off
*****
```

To do a 3-D plot on the x - t rectangle $0 < x < 2, 0 < t < 1$ you subdivide it into rectangles of side length dx by dt and plot the u at the vertices.

Here we choose $dx=.05$ and $dt=.05$.

Enter

```
*****
x=0:.05:2;
t=0:.05:1;
[x,t]=meshgrid(x,t);
mesh(x,t,U(x,t,50))
*****
```

ASSIGNMENT 7 :

1. HEAT EQUATION

Let u model the temperature in a rod and solve

$$d^2u/dx^2 = du/dt \quad 0 < x < 40, \quad t > 0$$

$$u(0,t) = u(40,t) = 0 \quad t > 0,$$

$$u(x,0) = x \quad \text{for } 0 < x < 40.$$

- Find the series solution and consider the partial sum $U = U(x,t,50)$.
- Plot U versus x for $t=5,10,20,40,100$ and 200 .
- For each value of t used in b) estimate the value of x for which the value of U is greatest. (See HW.2.) Plot these values versus t to see how the warmest point in the rod changes with time.
- Plot U versus t for $x=10,20$ and 30 . The command `fplot` only recognizes "x" as an independent variable. For example if you have an M-file for a function $g(x,t)$, and you want the graph of $g(3,t)$ for $1 < t < 3$ type :
`fplot('g(3,x)', [1,3])`.
- Do a 3-D plot of U versus x and t .
- How long does it take for the entire rod to cool off to a temperature of no more than 2 degrees ?

2. WAVE EQUATION

Let u model the displacement of a vibrating string and solve

$$d^2u/dx^2 = d^2u/dt^2 \quad 0 < x < 10, \quad t > 0$$

$$u(0,t) = u(10,t) = 0 \quad t > 0,$$

$$u(x,0) = \begin{cases} 1 & \text{for } 4 < x < 6 \\ 0 & \text{for } 0 < x < 4 \text{ \& } 6 < x < 10 \end{cases}$$

$$du/dt(x,0) = 0 \quad \text{for } 0 < x < 10.$$

- Find the series solution and consider the partial sum $U = U(x,t,50)$.
- Plot U versus x for $t=0,2,4,\dots,18,20$. Each on its own axes.
- Plot U versus t for $0 < t < 20$ for $x=5$ and $x=2.5$ on separate axes.
- Describe the motion of the string in a few sentences.