

MATLAB.8

Numerical Methods First consider the ode

$$y' = f(x, y) = 2xy^2$$

$$y(0) = 0.1.$$

We analyze this using the Euler, Improved Euler and Runge-Kutta methods. First we make a M-file for the right hand side.

```
*****
function w=f(x,y)
w=2*x*y^2;
*****
```

Next we make M-files for each of the three methods.

```
*****
function [X,Y]=eu(x,y,xf,n)
h=(xf-x)/n;
X=x;Y=y;
for i=1:n
    y=y+h*f(x,y);
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
```

```
*****
function [X,Y]=imeul(x,y,xf,n)
h=(xf-x)/n;
X=x;Y=y;
for i=1:n
    k1=f(x,y);
    k2=f(x+h,y+h*k1);
    y=y+h*(k1+k2)/2;
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
```

```
*****
function [X,Y]=rk(x,y,xf,n)
h=(xf-x)/n;
X=x;Y=y;
for i=1:n
    k1=f(x,y);
    k2=f(x+h/2,y+h*k1/2);
    k3=f(x+h/2,y+h*k2/2);
    k4=f(x+h,y+h*k3);
    y=y+h*(k1+2*k2+2*k3+k4)/6;
    x=x+h;
    X=[X;x];
    Y=[Y;y];
end
```

Here (x,y) are the initial values, xf is the final x-value and n is the number of partitions.

[X,Y] is the (n+1) x 2 matrix representing the computed nodes.

To plot all three you go to the command window and enter

```
*****
```

```
[z,w]=eu(0,0.1,3,20);
[s,t]=imeul(0,0.1,3,20);
[u,v]=rk(0,0.1,3,20);
plot(z,w,s,t,'--',u,v,'o')
```

Euler is given by a solid curve, Improved Euler by '--'s and Runge-Kutta by 'o's.

ASSIGNMENT 8 :

1. Let $f(x,y)$ be as above. Type in the M-files for f.m and the three approximations.

Plot their graphs for $n=20$ and $xf=3$.

The actual solution is $y(x)=1/(10-x^2)$.

To find the distance between $y(x)$ and the Euler approximation on the interval $[0,3]$ for a given value of n type :

```
x=0:3/n:3;
y=1./(10-x.^2);
y=y';
```

% Here we have transposed y from a row vector to a column vector.

```
[z,w]=eu(0,0.1,3,n);
max(abs(y-w))
```

Here

$\text{abs}(y-w)=[\text{abs}(y(1)-w(1)), \dots, \text{abs}(y(n+1)-w(n+1))]$ '
and $\max(\text{abs}(y-w))$ is the maximum of the $n+1$ components.

The theory predicts that

$$(*) \quad \max(\text{abs}(y-w)) < C * (3/n)$$

for some constant C and each $n > 1$.

2. Set $C1(n) = \max(\text{abs}(y-w)) / (3/n)$.

Compute $C1(n)$ for $n=100, n=200, \dots, n=800$.

Does $C1(n)$ grow as n gets large or tend to level off? Is this consistent with (*) ?

Set $C2(n) = \max(\text{abs}(y--t)) / (3/n)^2$ for the Improved Euler method.

Compute $C2(n)$ for the same values of n .

What should $C3(n)$ be for the Runge-Kutta method? Compute $C3(n)$.