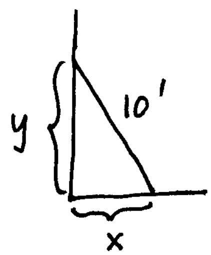
1. A ladder 10 feet long is leaning against a wall. The foot of the ladder is being pulled away from the wall at 3 feet per second. How fast, in feet per second, is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?



Know: $\frac{dx}{dt} = 3$

want: $\frac{dy}{dt}$ when $x = 8$. $(y=6)$

$x^2+y^2 = 100 \longrightarrow \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$\longrightarrow (8)(3) + (6)\left(\frac{dy}{dt}\right) = 0 \longrightarrow \frac{dy}{dt} = \frac{-24}{6} = -4.$

Top of ladder is sliding down wall at 4 (ft/sec)

A. 1
B. 2
C. 3
D. 4
E. 5

2. A spherical tank has radius equal to 10 feet (= 120 inches). Use differentials to estimate, in cubic inches, the amount of paint needed to cover the surface with a layer $\frac{1}{100}$ of an inch thick. ($V = \frac{4}{3}\pi r^3$).

$dV = \left(\frac{dV}{dr}\right)(dr) = (4\pi r^2)(dr)$

$dV\Big|_{r=120,\ dr=\frac{1}{100}} = 4\pi(120)^2\left(\frac{1}{100}\right) = 576\pi$

A. 288π
B. 480π
C. 576π
D. 640π
E. 960π

3. Find the absolute minimum of the function

$$f(x) = 4x^3 - 15x^2 + 12x + 7$$

on the closed interval $[0, 3]$.

$f'(x) = 12x^2 - 30x + 12 = 6(2x-1)(x-2)$

critical numbers are $x = \frac{1}{2}$ and $x = 2$.

x	f(x)
0	7
0.5	9.75
2	3 ← absolute minimum
3	16

A. 0
B. 1
C. 3
D. 5
E. 7

4. How many real roots does the equation $x^7 + x + 1 = 0$ have?

Let $f(x) = x^7 + x + 1$. Then $f'(x) = 7x^6 + 1$.
Note that $7x^2 + 1 > 0$ for all x.
Thus f is always increasing. Note that $f(-2) < 0$ and $f(2) > 0$.

Since f is increasing, $f(x) = 0$ must have one solution between -2 and 2. No solutions can exist for $x \le -2$ or $x \ge 2$.

A. 1
B. 2
C. 3
D. 5
E. 7

5. Find the largest interval on which the function $f(x) = x \sin x + \cos x$, $0 \le x \le \pi$, is increasing.

$f'(x) = \sin x + x\cos x - \sin x = x \cos x$

$\rightarrow$ sign of f is same as sign of $\cos x$.

$\cos(x)$ is positive on $(0, \pi/2)$

A. $(0, \pi)$
B. $(0, \frac{\pi}{2})$
C. $(\frac{\pi}{2}, \pi)$
D. $(0, \frac{\pi}{3})$
E. $(\frac{\pi}{3}, \frac{5\pi}{3})$

6. What is the length of the largest interval on which the function $f(x) = x^3 - 3x^2 - 9x$ is decreasing?

$f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$

	$-\infty$ to -1	-1 to 3	3 to ∞
$x-3$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
$f'(x)$	$+$	$-$	$+$

f decreasing on $(-1, 3)$, which has length 4

A. 1
B. 2
C. 3
D. 4
E. ∞

7. On what interval is the graph of the function

$$f(x) = 1 - \frac{2}{x} + \frac{1}{x^2}$$

concave downward?

$$f'(x) = \frac{2}{x^2} - \frac{2}{x^3}, \quad f''(x) = \frac{-4}{x^3} + \frac{6}{x^4} = \frac{-4x+6}{x^4}$$

$$f''(x) < 0 \rightarrow -4x+6 < 0 \rightarrow -4x < -6 \rightarrow x > \frac{6}{4}$$

A. $(\frac{3}{2}, \infty)$
B. $(1, \frac{3}{2})$
C. $(-\infty, 0)$
D. $(0, 1)$
E. $(1, \infty)$

8. $\lim_{x\to\infty} \frac{(\ln x)^3}{x^2} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{3(\ln x)^2(\frac{1}{x})}{2x} = \lim_{x\to\infty} \frac{3(\ln x)^2}{2x^2} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{6(\ln x)(\frac{1}{x})}{4x} = \lim_{x\to\infty} \frac{6\ln x}{4x^2} = \frac{\infty}{\infty}$$

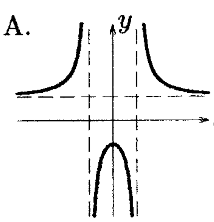
$$\lim_{x\to\infty} \frac{6(\frac{1}{x})}{8x} = 0.$$

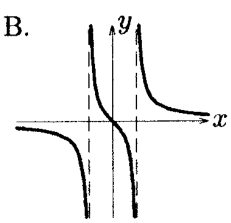
A. 0
B. 1
C. $\frac{3}{2}$
D. $\frac{9}{4}$
E. ∞

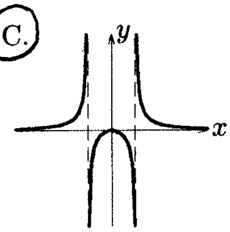
9. Given the following information about limits, select the graph that could be the graph of $y = f(x)$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to-1^-} f(x) = \lim_{x\to 1^+} f(x) = \infty$$

$$\lim_{x\to-1^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$$

A.


B.


C.


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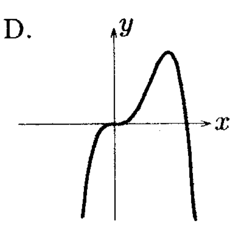
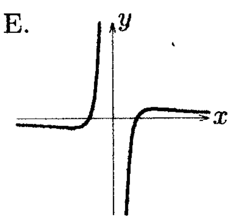
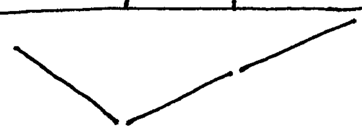


E.


10. The function $f(x) = x^4 - 3x^3 + 3x^2 - x$ has critical numbers $c = \frac{1}{4}, 1$; indeed $f'(x) = (4x-1)(x-1)^2$. At these critical numbers f has

	$-\infty$ to $\frac{1}{4}$	$\frac{1}{4}$ to 1	1 to ∞
$4x-1$	$-$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$
$f'(x)$	$-$	$+$	$+$

f



A. a local max. at $\frac{1}{4}$, a local min. at 1

B. a local max. at 1, a local min. at $\frac{1}{4}$

C. a local max. at 1, neither a local max. nor a local min. at $\frac{1}{4}$

D. a local min. at $\frac{1}{4}$, neither a local max. nor a local min. at 1

E. neither a local max. nor a local min. at either $\frac{1}{4}$ or 1

local min at $x = \frac{1}{4}$

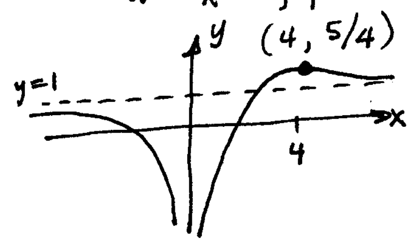
neither local max nor local min at $x = 1$

11. Find the maximum value of the function $\frac{x^2+2x-4}{x^2}$. $= f(x)$

$f(x) = 1 + \frac{2}{x} - \frac{4}{x^2}$

$f'(x) = -\frac{2}{x^2} + \frac{8}{x^3} = \frac{-2x+8}{x^3}$. critical numbers are $x = 0, 4$

	$-\infty$ to 0	0 to 4	4 to ∞
$-2x+8$	$+$	$+$	$-$
x^3	$-$	$+$	$+$
$f'(x)$	$-$	$+$	$-$



A. $\frac{1}{4}$

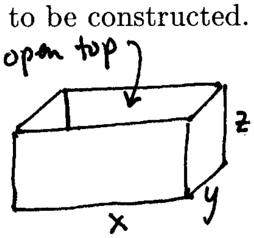
B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. $\frac{3}{4}$

E. $\frac{5}{4}$

12. A rectangular cardboard box of 32 in^3 volume with a square base and an open top is to be constructed. Neglecting waste, find the minimum area of cardboard needed.



Volume $= 32 = xyz$

square base $\to x = y \to 32 = x^2 z$

$\to z = 32/x^2$

A. 54 in^2

B. 48 in^2

C. 46 in^2

D. 42 in^2

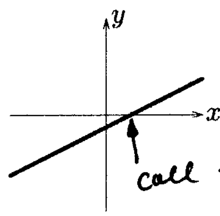
E. 40 in^2

Surface area $= A = x^2 + 4xz$ (no top!)

$A(x) = x^2 + 4x\left(\frac{32}{x^2}\right) = x^2 + \frac{128}{x}$

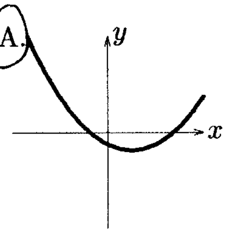
$A'(x) = 2x - \frac{128}{x^2} = \frac{2x^3 - 128}{x^2} = 0 \to x = 4$. $A(4) =$

13. Given the graph of $y = f'(x)$ below, select a graph which could be the graph of $y = f(x)$.

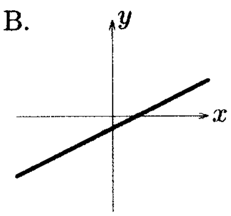


call this point $x = a$.

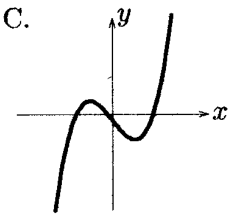
A.



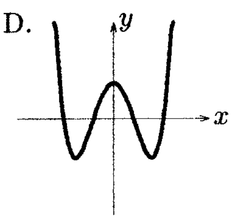
B.



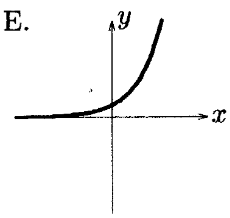
C.



D.



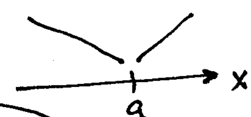
E.



Note: $f'(x) < 0$ for $x < a$

$f'(x) > 0$ for $x > a$

$\rightarrow$ graph of f must be something like



$\Rightarrow$ A. must be the answer

14. If $f''(x) = 12x^2 + 2$, $f(0) = 2$ and $f'(0) = 3$, find $f(1)$.

A. 3
B. 4
C. 5
D. 6
E. 7

$\rightarrow f'(x) = 4x^3 + 2x + C_1$

$f'(0) = 0 + 0 + C_1 = 3 \rightarrow C_1 = 3$

$\rightarrow f'(x) = 4x^3 + 2x + 3$

$\rightarrow f(x) = x^4 + x^2 + 3x + C_2$

$f(0) = 0 + 0 + 0 + C_2 = 2 \rightarrow C_2 = 2$

$\rightarrow f(x) = x^4 + x^2 + 3x + 2$

$\rightarrow f(1) = 1 + 1 + 3 + 2 = 7$