Homework #35, pg. 739

Does $\sum \left(\frac{n}{n+1}\right)^{n^2}$ converge.

USE ROOT TEST

$$\sqrt[n]{a_n} = \left(\left(\frac{n}{n+1}\right)^{n^2}\right)^{1/n} = \left(\frac{n}{n+1}\right)^n$$

Take $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\text{[illegible]}\right)^n$.

In Chapter ~~4~~ you saw

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$, which

I show you again:

$0'$

Take $f(x) = \left(1+\frac{1}{x}\right)^x$

$= e^{x \ln(1+\frac{1}{x})}$

Since $F(x) = e^x$ is continuous

(*) $\lim_{x\to\infty} e^{x\ln(1+\frac{1}{x})} = e^{\lim_{x\to\infty} x\ln(1+1/x)}$.

$\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

$1+\frac{1}{x} \longrightarrow 1$, so $\ln\left(1+\frac{1}{x}\right) \longrightarrow \ln(1)$

$= 0$

so $x\ln\left(1+\frac{1}{x}\right) \longrightarrow \frac{0}{0}$?

Use L'Hopital's rule:

$\lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{x}}\right)\cdot\frac{-1}{x^2}}{-1/x^2}$

$= \lim_{x\to\infty} \left(\frac{1}{1+1/x}\right) = 1$ go back to

(*) $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e^1 = e$.