MA 222 **Final Exam Practice Problems**

The Table of Integrals (pages 481-484 of the text) and the Formula Page may be used. They will be attached to the final exam.

1. Calculate $\sec 135°$ correct to 4 decimal places.
   A. -0.7071 B. -0.9961 C.-1.0039 D. -1.4142 E. None of these.

2. Calculate $\dfrac{dy}{dx}$ if $y = \cos(1-2x)$.
   A. $-\sin(1-2x)$ B. $-2\sin(1-2x)$ C. $2\sin(1-2x)$ D. $\sin(1-2x)$ E. $-2\cos(1-2x)$

3. Find y' if $y = x\tan^2 x$.
   A. $2x\tan x + \tan^2 x$ B. $2x\tan x\sec^2 x$ C. $x\sec^2 x + \tan^2 x$ D. $2x\tan x\sec^2 x + \tan^2 x$
   E. None of these.

4. If $\sin\theta = -0.5473$ and $\pi < \theta < \frac{3\pi}{2}$, find θ in radians. Give your answer correct to 4 decimal places.
   A. -0.5791 B. 3.7207 C. 3.1511 D. 2.5625 E. 1.761

5. Express as a single logarithm: $\log x^3 - \log\sqrt{x}$.
   A. $\log(x^3 - \sqrt{x})$ B. $\log(\frac{5}{2}x)$ C. $\log(x^6)$ D. $\log(3x - \frac{x}{2})$ E. $\log(x^{\frac{5}{2}})$

6. If $y = e^{x^2}$ calculate y'.
   A. $2xe^{x^2}$ B. e^{2x} C. $x^2e^{x^2-1}$ D. $2xe^{2x}$ E. e^{x^2}

7. If $y = \ln\sqrt{x^2+1}$ calculate y'.
   A. $\dfrac{1}{\sqrt{x^2+1}}$ B. $\dfrac{2x}{\sqrt{x^2+1}}$ C. $\dfrac{x}{x^2+1}$ D. $\dfrac{1}{2(x^2+1)}$ E. None of these.

8. Find an equation for the tangent line to the curve $e^y + x^2 = 2$ at the point $(1,0)$.
   A. $y = x-1$ B. $y = 2x-2$ C. $y = -2x+2$ D. $y = -x+1$ E. $y = -2x-2$

9. Find the maximum value of the function $f(x) = x^2\log(\frac{2}{x})$.
   A. 1 B. e^2 C. $2e$ D. 2 E. $2/e$

10. Which of the following best describes the function $y = \ln x - x$?
    A. There is a relative minimum at $x = 1$ and the curve is concave down for all $x > 0$.
    B. There is a relative maximum at $x = 1$ and the curve is concave down for all $x > 0$.
    C. There is a relative maximum at $x = 1$, the curve is concave down for $0 < x < 1$, and concave up for $x > 1$.
    D. There is a relative minimum at $x = 1$, the curve is concave down for $0 < x < 1$, and concave up for $x > 1$.
    E. None of these.

11. The velocity of an object falling through a resisting medium is given by $v = 100(1 - e^{-0.001t})$. Find the acceleration when $t = 100$. Give your answer correct to two decimal places.
    A. 0.09 B. 9.52 C. 90.48 D. 0.38 E. 1.14

12. Find y' if $y = x\cos 2x$.
    A. $-x\sin 2x + \cos 2x$ B. $-2x\sin 2x + \cos 2x$ C. $x\sin 2x + \cos 2x$ D. $2x\sin 2x + \cos 2x$
    E. $-2\sin 2x + \cos 2x$

13. Evaluate $\displaystyle\int \frac{xdx}{\sqrt{1-x^2}}$.
    A. $x\ln|1-x^2|+C$ B. $2\sqrt{1-x^2}+C$ C. $-\frac{1}{2}\ln|1-x^2|+C$ D. $-\sqrt{1-x^2}+C$ E. None of these.

14. Evaluate $\int_1^2 \frac{dx}{\sqrt{9x^2-4}}$. (Give your answer correct to 3 decimal places.)
A. 0.800 B. 0.267 C. 2.401 D. 0.928 E. 0.743

15. Evaluate $\int_1^3 \sqrt{x}\ln x dx$. (Give your answer correct to 2 decimal places.)
A. 1.94 B. 1.50 C. -0.21 D. 1.01 E. 1.27

16. Evaluate $\int (\sin^5 3x)\,dx$ using a reduction formula.
A. $-\frac{1}{15}(\sin^4 3x)(\cos 3x) - \frac{1}{9}(\cos 3x)(\sin^2 3x + 2) + C$ B. $-\frac{1}{18}(\cos^6 3x) + C$
C. $-\frac{1}{15}(\sin^4 3x)(\cos x) + \frac{3}{10}x - \frac{1}{15}\sin 6x + \frac{1}{120}\sin 12x + C$
D. $-\frac{1}{15}(\sin^4 3x)(\cos 3x) - \frac{4}{45}(\cos 3x)(\sin^2 3x + 2) + C$ E. None of these.

17. Find the area of the region bounded by the graph of $y = \sin 2x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{2}$.
A. 2 B. 1 C. 0 D. $\frac{1}{2}$ E. $\frac{3}{4}$

18. Find the first three non-zero terms of the Maclaurin series of $f(x) = \sqrt{1+3x}$.
A. $f(x) = 1 + \frac{3}{2}x - \frac{9}{4}x^2$ B. $f(x) = 1 + \frac{1}{2}\sqrt{1+3x} - \frac{1}{8}(1+3x)$ C. $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$
D. $f(x) = 1 + \frac{3}{2}\sqrt{1+3x} - \frac{9}{8}(1+3x)$ E. $f(x) = 1 + \frac{3}{2}x - \frac{9}{8}x^2$

19. Using the Maclaurin series $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \ldots$, find the minimum number of terms required to calculate $\ln(1.3)$ so that the error is ≤ 0.001.
A. 2 B. 3 C. 4 D. 5 E. 6

20. Find the first three non-zero terms in the Taylor series for $f(x) = \sin 2x$ in powers of $(x - \frac{\pi}{8})$.
A. $f(x) = \sqrt{2}[\frac{1}{2} + (x-\frac{\pi}{8}) - (x-\frac{\pi}{8})^2]$ B. $f(x) = 2(x-\frac{\pi}{8}) - \frac{3}{2}(x-\frac{\pi}{8})^2 + \frac{4}{15}(x-\frac{\pi}{8})^5$
C. $f(x) = (x-\frac{\pi}{8}) - \frac{1}{3!}(x-\frac{\pi}{8})^3 + \frac{1}{5!}(x-\frac{\pi}{8})^5$ D. $f(x) = \sqrt{2}[\frac{1}{2} + \frac{1}{2}(x-\frac{\pi}{8}) - \frac{1}{4}(x-\frac{\pi}{8})^2]$
E. None of these.

21. Approximate $\int_0^{0.3} \cos\sqrt{x}dx$ using three terms of the appropriate Maclaurin series. (Give your answer correct to 4 decimal places.)
A. 0.8538 B. 0.2779 C. 0.9553 D, 0.2955 E. 0.1863

22. If f is a periodic function of period 2π and

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < 0 \\ 1 & \text{for } 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} < x \leq \pi \end{cases}$$

calculate the first three non-zero terms of the Fourier series for $f(x)$. (That is, the first three non-zero terms in the series: $a_0 + a_1\cos x + b_1 \sin x + a_2\cos 2x + b_2\sin 2x + \cdots$)
A. $\frac{\pi}{4} + \cos x + \sin x$ B. $\frac{1}{4} + \frac{1}{\pi}\cos x - \frac{1}{\pi}\sin x$. C. $\frac{1}{4} - \frac{\sqrt{2}}{\pi}\cos x + \frac{1}{\pi}\cos 2x$
D. $\frac{1}{4} + \frac{1}{\pi}\cos x + \frac{1}{\pi}\sin x$ E. None of these.

23. Find the general solution of the differential equation $y^2dx + (x+1)^2dy = 0$.
A. $\frac{1}{3}(x+1)^3 + \frac{1}{3}y^3 = C$ B. $\frac{1}{x+1} + \frac{1}{y} = C$ C. $\ln|x+1| + \ln|y| = C$
D. $2(x+1) + 2y = C$ E. $x + \frac{1}{y} = C$

24. Find the particular solution of the differential equation $y' + \frac{1}{x}y = x^2$ where $y = 2$ when $x = 1$.
A. $y = \frac{x^4}{4} + \frac{7}{4}$ B. $y = \frac{x^3}{3} + \frac{5}{3x}$ C. $y = \frac{x^3}{4} + \frac{7}{4x}$ D. $y = \frac{x^3}{4} + \frac{7}{4}$ E. None of these.

25. Find the particular solution of the differential equation $y'' + y' - 6y = 0$ where $y' = 0$ and $y = -1$ when $x = 0$.
A. $y = -\frac{1}{5}(2e^{-3x} + 3e^{2x})$ B. $y = -\frac{1}{5}(2e^{3x} + 3e^{-2x})$ C. $y = -\frac{1}{2}(e^{-3x} + e^{2x})$
D. $y = -\frac{1}{2}(e^{3x} + e^{-2x})$ E. None of these.

26. Find the general solution of the differential equation $D^2y - Dy + y = 0$.
A. $y = c_1e^{(1+\sqrt{3})x/2} + c_2e^{(1-\sqrt{3})x/2}$ B. $y = e^x[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$
C. $y = e^x[c_1\sin(\sqrt{3}x) + c_2\cos(\sqrt{3}x)]$ D. $y = e^{x/2}[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$
E. None of these.

27. Find the equation of the orthogonal trajectories of the curves $y = cx^5$.
A. $15cx^3y = 1$ B. $x^2 + 5y^2 = c$ C. $y = \frac{1}{15x^3} + c$ D. $\frac{1}{5}\ln|y| + \ln|x| = c$ E. $5cyx^4 = -1$.

28. Find the equation of the curve for which the slope at any point (x, y) is $x + y$ and which passes through the point $(0, 1)$.
A. $y = 2e^{-x} - x - 1$ B. $y = \frac{1}{2}e^x + \frac{1}{2}x^2$ C. $y = -x + 1$ D. $y = 2e^x - x - 1$ E. $y = e^x + x$

29. An object moves with simple harmonic motion according to the equation $\frac{d^2x}{dt^2} + 64x = 0$. Find the displacement x as a function of t if $x = 4$ and $\frac{dx}{dt} = 3$ when $t = 0$.
A. $x = 4\sin 8t + \frac{3}{8}\cos 8t$ B. $x = 3\sin 8t + 4\cos 8t$ C. $x = \frac{3}{64}\sin 64t + 4\cos 64t$
D. $x = \frac{3}{8}\sin 8t + 4\cos 8t$ E. $x = 8\sin 8t + 4\cos 8t$

30. Find the general solution of the differential equation $D^2y + 8Dy + 16y = 0$.
A. $y = c_1e^{-4x} + c_2xe^{-4x}$ B. $y = c_1e^{4x} + c_2xe^{4x}$ C. $y = c_1e^{-4x} + c_2e^{-4x}$ D. $y = c_1\sin 4x + c_2\cos 4x$
E. $y = c_1e^{4x} + c_2e^{-4x}$

31. Calculate the Laplace transform of $2e^{-3t}\sin 4t$.
A. $\frac{2}{(s-3)^2+16}$ B. $\frac{8}{(s+3)^2+16}$ C. $\frac{8}{(s-3)^2+16}$ D. $\frac{8}{(s+3)(s^2+16)}$ E. $\frac{2}{(s+3)^2+16}$

32. Calculate the inverse Laplace transform of $\frac{2s}{s^2+3s-4}$.
A. $\frac{1}{10}(4e^{4t} - e^t)$ B. $\frac{2}{5}(4e^{-4t} + e^t)$ C. $\frac{1}{10}(4e^{4t} + e^{-t})$ D. $\frac{2}{5}(4e^{4t} + e^{-t})$ E. None of these

33. Calculate the Laplace transform of the expression: $y'' - 3y' + 2y$, where $y = f(x)$, $f(0) = -1$ and $f'(0) = 2$.
A. $(s^2 - 3s + 2)L(f)$ B. $s^2L(f) + s - 2$ C. $(s^2 - 3s + 2)L(f) + s - 1$ D. $(s^2 - 3s + 2)L(f) + s + 1$
E. $(s^2 - 3s + 2)L(f) + s - 5$

34. Find the Laplace transform of the solution of the differential equation: $y' + 2y = e^{-2t}$; $y(0) = 2$.
A. $\frac{1}{(s+2)^2}$ B. $2 + \frac{1}{s+2}$ C. $\frac{2}{s+2} + \frac{1}{(s+2)^2}$ D. $\frac{2}{s-2} + \frac{1}{(s-2)^2}$ E. $\frac{1}{(s-2)^2}$

35. Use Laplace transforms to solve the differential equation: $y'' + 9y = 3t$; $y(0) = 1, y'(0) = -1$.
A. $y = \frac{1}{3}t - \frac{4}{9}\sin 3t + \cos 3t$ B. $y = \frac{1}{9}t - \frac{10}{27}\sin 3t + \cos 3t$
C. $y = 4\cos 3t - \frac{1}{3}\sin 3t$ D. $y = \cos 3t - \frac{1}{3}\sin 3t$ E. None of these.

36. Use Laplace transforms to solve the differential equation $D^2y - 2Dy + y = e^t$; $y(0) = 0, y'(0) = 0$.

A. $y = 2t^2e^t$ B. $y = \frac{1}{2}t^2e^{-t}$ C. $y = \frac{1}{2}t^2e^t$ D. $y = t^2e^{-t}$ E. $y = 2te^{-t}$

37. If $f(s) = \dfrac{s}{(s-1)^2(s+2)}$, which of the following is the partial fraction expansion of $f(s)$? (A, B and C are constants.)
A. $\dfrac{A}{s-1} + \dfrac{B}{s-1} + \dfrac{C}{s+2}$ B. $\dfrac{A}{(s-1)^2} + \dfrac{B}{s+2}$ C. $\dfrac{As}{s-1} + \dfrac{Bs}{(s-1)^2} + \dfrac{Cs}{s+2}$
D. $\dfrac{A}{s-1} + \dfrac{B}{(s-1)^2} + \dfrac{C}{s+2}$ E. $\dfrac{A}{s-1} + \dfrac{B}{s+2}$

38. Approximate $\displaystyle\int_0^2 \frac{dx}{x^2+2}$ to 2 decimal places using the Trapezoidal Rule with $n = 4$.
A. 1.35 B. 0.87 C. 0.67 D. 2.50 E. 1.33

39. Approximate $\displaystyle\int_0^1 \frac{dx}{x+1}$ to 3 decimal places using Simpson's Rule with $n = 4$.
A. 0.697 B. 1.817 C. 2.772 D. 2.079 E. 0.693

40. If the current in an AC circuit is given by $i = \cos t + \sin t$, then the first maximum of the current after $t = 0$ is
A. 2 A B. $\frac{1}{\sqrt{2}}$ A C. 1 A D. $\sqrt{2}$ A E. $\frac{1}{2}$ A

41. A certain radioactive substance decays according to the law $N = 6e^{-2t}$, where N (in kilograms) is the amount present and t is the time in years. Find the time rate of change of N with respect to t when $t = 2$, rounded to the nearest hundredth.
A. -0.22 B. -0.02 C. 0.02 D. 0.22 E. -0.012

42. Find the current, i, as a function of time, t, for a LC circuit with $L = 1$ H and $C = 1.0 \times 10^{-4}$ F, if you know that $e(t) \equiv 0$, and $i = 10$ and $q = 0$ when $t = 0$.
A. $100\cos 10t$ B. $10\cos 100t$ C. $0.1\sin 100t$ D. $100\sin 10t$ E. $10\sin 100t$

Answers

1. D; 2. C; 3. D; 4. B; 5. E; 6. A; 7. C; 8. C; 9. E; 10. B; 11. A; 12. B; 13. D; 14. B; 15. A; 16. D; 17. B 18. E; 19. C; 20. A; 21. B; 22. D; 23. B; 24. C; 25. A; 26. D; 27. B; 28. D; 29. D; 30. A; 31. B; 32. B; 33. E; 34. C; 35. A; 36. C; 37. D; 38. C; 39. E; 40. D; 41. A; 42. B.