

## MA 224 FORMULAS

### THE SECOND DERIVATIVE TEST

Suppose  $f$  is a function of two variables  $x$  and  $y$ , and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose  $(a, b)$  is a critical point of  $f$ .

1. If  $D(a, b) < 0$ , then  $f$  has a saddle point at  $(a, b)$ ,
2. If  $D(a, b) > 0$  and  $f_{xx}(a, b) < 0$ , then  $f$  has a relative maximum at  $(a, b)$ .
3. If  $D(a, b) > 0$  and  $f_{xx}(a, b) > 0$ , then  $f$  has a relative minimum at  $(a, b)$ .
4. If  $D(a, b) = 0$ , the test is inconclusive.

### LEAST-SQUARES LINE

The equation of the least-squares line for the  $n$  points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , is  $y = mx + b$ , where

$$m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}, \quad b = \frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}.$$

### TRAPEZOIDAL RULE

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} \left[ f(x_1) + 2f(x_2) + \dots + 2f(x_n) + f(x_{n+1}) \right],$$

$$\text{where } x_{i+1} - x_i = \Delta x = \frac{b-a}{n}, \quad x_1 = a, \quad x_{n+1} = b.$$

### ERROR ESTIMATE FOR THE TRAPEZOIDAL RULE

If  $M$  is the maximum value of  $|f''(x)|$  on the interval  $a \leq x \leq b$ , then

$$|E_n| \leq \frac{M(b-a)^3}{12n^2}$$

### PROBABILITY

If  $f(x)$  is a probability density function, then

$$\text{Expected Value (Mean)} = E = \int_{-\infty}^{\infty} xf(x) dx$$

$$\text{Variance} = V = \int_{-\infty}^{\infty} x^2 f(x) dx - E^2$$

### GEOMETRIC SERIES

If  $|r| < 1$ , with  $r \neq 0$ , then

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}.$$

### TAYLOR SERIES

The Taylor series of  $f(x)$  about  $x = a$  is the power series  $\sum_{n=0}^{\infty} a_n(x-a)^n$ , where

$$a_n = \frac{f^{(n)}(a)}{n!}.$$

Examples: (with  $a = 0$ )

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \text{for } -\infty < x < \infty; \quad \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n, \quad \text{for } -1 < x \leq 1.$$