MA 387 — Assignment Sheet — Fall 2002

Text: Set Theory and Logic by Robert R. Stoll, Dover Publications, Inc.

1. Read §1.1, pp 1-3 and the "Historical Introduction" handout.
2. Read §1.2-3. p9: 2.2, 2.4, 2.5aegh; p12: 3.2c, 3.3, 3.4abc, 3.8, 3.9.
3. Read §1.4. p15: 4.2, 4.3a, 4.4, 4.6abc, 4.7, 4.8, 4.9abc, 4.11a.
4. Read §1.5. p22: 5.3bc, 5.8ab,
   For each of the following, if it is true give a proof; if not, give a counterexample.

(i) $B \subseteq C \to A - C \subseteq A - B$.

(ii) $A \cap B = A \cap C \to B = C$.

(iii) $(A - B) - C = A - (B - C)$.

(iv) $(A - \overline{B}) \cup (\overline{B} - A) = \overline{(\overline{A} \cap B)} - \overline{(\overline{A} \cup B)}$.

(v) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.

(vi) $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

5. Read §1.6. p28: 6.2, 6.3, 6.8, 6.9, 6.12.
6. Read §1.7. p33: 7.1, 7.3, 7.10, 7.11.
7. Read §1.8. p37: 8.1, 8.4, 8.8, 8.9, 8.10.
8. Read §1.9. p42: 9.1, 9.10bd, 9.13, 9.15c, 9.16a.
9. Read §1.10. p47: 10.2ab, 10.5, 10.6a.

(i) Prove that for every set A, $\bigcup\bigcup(A \times A) = A$.

(ii) Prove that for every $a \in A$, $\bigcap A \subseteq a \subseteq \bigcup A$.

(iii) Calculate each of the following:

  (a) $\bigcap_{n\in\mathbb{Z}^+}(0, \frac{1}{n})$

  (b) $\bigcap_{n\in\mathbb{Z}^+}[0, \frac{1}{n})$

  (c) $\bigcup_{n\in\mathbb{Z}^+}(0, 1 + \frac{1}{n})$

where $(0, \frac{1}{n})$, $[0, \frac{1}{n})$, and $(0, 1 + \frac{1}{n})$ are each intervals of real numbers.

10. Read §1.11. p54: 11.5, 11.9, 11.13, 11.14, 11.18.

(i) Define a relation R on $\mathbb{Z}$ as follows: If $a, b \in \mathbb{Z}$, aRb iff

  (a) a and b are odd and $a \leq b$.

  (b) a and b are even and $a \leq b$.

  (c) a is odd and b is even.

List the elements of $\mathbb{Z}$ as ordered by R and show that $\mathbb{Z}$ is linearly ordered by R, but not well ordered.

(ii) Define a relation S on $\mathbb{N} \times \mathbb{N}$ as follows: If $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$, $(a, b)S(c, d)$ iff

  (a) $a < c$.

  (b) $a = c$ and $b \leq d$.

List the elements of $\mathbb{N} \times \mathbb{N}$ as ordered by S and show that S well orders $\mathbb{N} \times \mathbb{N}$.

(OVER)

11. Read §2.1-2.
(i) Prove by induction that for each $n \in \mathbb{N}^+$:
(a) $1 + 3 + 5 + \cdots + (2n-1) = n^2$.
(b) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} \cdots \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$.
(c) $1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots \cdot n > n^3$, for $n \geq 6$.
(ii) Give inductive definitions for the following functions:
(a) $n!$, where $0! = 1$.
(b) 2^n.
(c) $\min(m, n)$, for fixed m.
(d) $[\frac{n}{m}] =$ greatest integer $\leq \frac{n}{m}$, where m is a fixed positive integer.
(e) The function $L(n)$, where if there are $n \geq 2$ points in a plane, no three of which are colinear, $L(n)$ is the number of lines that can be drawn through two of the given n points. Also, prove that $L(n) = \frac{n(n-1)}{2}$.
12. Read §2.3. p86: 3.3, 3.5, 3.9, 3.10,
Show that for all sets A and B, if $(A-B) \sim (B-A)$, then $A \sim B$, but show that the converse (if $A \sim B$, then $(A-B) \sim (B-A)$) is false by constructing a counterexample.
13. Read §2.4. p94: 4.11, 4.12, 4.13.
14. Read §2.5. Cardinalities of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.

Let $\overline{\overline{\mathbb{R}}} = c$ and $\overline{\overline{\mathbb{R}^{\mathbb{R}}}} = f$. For each of the following sets, state whether its cardinal number is $\aleph_0$, c, f, or none of these and give reasons for your answer.
($\mathbb{A} = \{g |\ g : (0,1) \to \mathbb{R}\}$, $\mathbb{B} = \{g |\ g : (0,2) \to \mathbb{R}\}$.)
(a) $\mathbb{A}$.
(b) $(0,1) \cap \mathbb{Q}$.
(c) $\mathbb{N} \times (0,1)$.
(d) $\{g |\ g : \mathbb{Q} \to \mathbb{Z}\}$.
(e) $\{g |\ g : \mathbb{N} \to \mathbb{R} \times \mathbb{R}\}$.
(f) $\{g |\ g : \mathbb{N}^{\mathbb{N}} \to \mathbb{N}\}$.
(g) $\{g |\ g : \mathbb{Q} \to \{0,1\}\}$.
(h) $\{g |\ g : \{0,1\} \to \mathbb{Q}\}\}$.
(i) $\{g |\ g : \mathbb{Q} \to \mathbb{R}\}$.
(j) $\mathbb{A} \cup \mathbb{B}$.
(k) $\mathbb{A} \times \mathbb{B}$.
(l) $\mathbb{A}^{\mathbb{B}}$.