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1. (15) Let  $A = \{1, 2, 3\}$  and  $B = \{3, 4\}$ . Find each of the following sets.
- (a)  $A \cap \overline{B}$ .

(b)  $\mathcal{P}(A)$ .

(c)  $A \times B$ .

2. (10) Prove that for all sets  $A$ ,  $B$ , and  $C$ , if  $B \subseteq C$  then  $A - C \subseteq A - B$ .

3. (15) Prove that for all sets  $A$  and  $B$ ,

$$(A - \overline{B}) \cup (\overline{B} - A) = \overline{(A \cap B)} - \overline{(A \cup B)}.$$

4. (10) Show that the following statement is NOT a theorem of set theory: For all sets  $A$ ,  $B$ , and  $C$ , if  $B \subseteq \overline{C \cup A}$  and  $C \subseteq \overline{B \cup A}$  then  $C = \emptyset$ .

5. (10) If  $f(x) = 7\sqrt{x^2 + 1}$ , express  $f$  as the composition of four functions, none of which is the identity function.

6. (15) Show that if  $R$  is an equivalence relation, then  $R \circ R^{-1} = R$ .

7. (15) Prove that for all sets  $A$  and  $B$ ,  $\bigcap \bigcup \langle A, B \rangle = A \cap B$ .

8. (10) Suppose that  $A$  is a set of functions that is linearly ordered by the subset relation,  $\subseteq$ , prove that  $\bigcup A$  is a function.