

Name: _____

(20) 1. Prove that for all sets A, B and C,

$$C \subseteq A \cap B \text{ iff } (C \cap \bar{A}) \cup (C \cap \bar{B}) = \phi.$$

(10) 2. Show that for all sets A and B , $A^B \subseteq \mathcal{P}(B \times A)$.

(15) 3. Define a relation ρ on \underline{Z} such that for all $a, b \in \underline{Z}$
 $a \rho b$ iff $a - b$ is divisible by 5.

Show that ρ is an equivalence relation and describe the equivalence classes.

(35) 4. Suppose $f : \underline{\mathbb{R}} \rightarrow \underline{\mathbb{R}}$ such that for each $x \in \underline{\mathbb{R}}$, $f(x) = 3 + 2x$.

(a) Show f is a bijection (1-1 and onto).

(b) Let $(0,1) = \{x \in \underline{\mathbb{R}} \mid 0 < x < 1\}$. Calculate $f[(0,1)]$ and $f^{-1}[(0,1)]$.

(c) If $g : \underline{\mathbb{R}} \rightarrow \underline{\mathbb{R}}$ such that for each $x \in \underline{\mathbb{R}}$, $g(x) = x^2$, calculate $f \circ g(2)$ and $g \circ f(2)$.

(20) 5. If $f \subseteq A \times B$, prove that f is a function iff $f \circ f^{-1} \subseteq i_B$.