Math 387 Exam II April 18, 1975

Name ______________________________

In what follows, N is the set of natural numbers, Q is the set of rational numbers, and R is the set of real numbers.

(20) 1. Prove by mathematical induction that for all $n \in \underline{N}$, $n \geq 1$,

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n} > \frac{2}{5}.$$

2. For each $n,m \in N$, $m \neq 0$, let $rm(n,m)$ be the remainder when n is divided by m. Define a relation R on N as follows: if $n, m \in N$

$$nRm \text{ iff } rm(n,3) < rm(m,3), \text{ or}$$

$$rm(n,3) = rm(m,3) \text{ and } n \leq m.$$

(10) (i) Give an inductive definition of $rm(n,m)$.

(10) (ii) List the elements of N as ordered by R.

(10) (iii) Give the order type of $\langle N, R \rangle$.

(20) 2 (iv) Prove that R well orders N.

(30) 3. Each of the following sets have cardinal number $\aleph_0$, c, or f. State which it is.

(i) The set of all points in 3-dimensional space. (Ie. $\{\langle u, v, w\rangle \mid u, v, w \in \underline{R}\}$.)

(ii) The set of all $u \in \underline{R}$ which have the property that $u^n \in \underline{Q}$ for some $n \in \underline{N}$, $n \geq 1$.

(iii) The set of all functions which map the unit interval (0,1) onto itself. ($(0,1) = \{u \in \underline{R} \mid 0 < u < 1\}$.)

iv) The set of all constant real valued functions. (Ie. $\{g \mid (\exists u \in \underline{R}) g : \underline{R} \to \{u\}\}$.)

v) The set of all functions from $\underline{R}$ into $\underline{Q}$.

vi) The set of all functions from $\underline{Q}$ into $\underline{R}$.