MA 387 Exam 2 April 16, 1982

Name ______________________________

(10) 1. Calculate each of the following:

a) $\cap\{(a,b) \mid a < 0 \text{ and } b \geq 2\}$

b) $\cup\{(a,b) \mid [a,b] \subseteq (1,2)\}$.

(15) 2. Prove that $f[\bigcap_{i\in I} A_i] \subseteq \bigcap_{i\in I} f[A_i]$

(25) 3. Define a relation $\rho$ such that for all sets A and B,

$$A \rho B \quad \text{iff} \quad A = A \cup B$$

a) Prove that $\rho$ is a partial ordering relation.

b) Give an example of an infinite set x such that $\langle x, \rho \rangle$ is a well ordered set, where $\rho$ is defined above.

(25) 4. Consider the sequence

$$x_1 = \sqrt{2},\ x_2 = \sqrt{2 + \sqrt{2}},\ x_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}},\ \ldots$$

a) Give an inductive definition of $x_n$.

b) Prove that $x_n < 2$ for all $n \geq 1$.

(10) 5. Prove that for all sets A, if $N \precsim A$ and $x \notin A$ then $A \cup \{x\} \approx A$.

(15) 6. Prove using mathematical induction that

$$(1 - x)(1 + x)(1 + x^2)(1 + x^4)\cdots(1 + x^{2^n}) = (1 - x^{2^{n+1}})$$

for all $n \geq 0$.