

Name \_\_\_\_\_ Stud. No. \_\_\_\_\_

1. (20) If  $A = \{a, b, c\}$  and  $B = \{0, 1\}$  calculate  $A \times B$  and  $\mathcal{P}(A)$ .

2. (20) Prove that for all sets  $A$ ,  $B$ , and  $C$ ,  $(A \cup B) - C = (A - C) \cup (B - C)$ .

3. (30) Let  $n \in \mathbb{N}^+$  and let  $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ . Calculate each of the following.

(a)  $\bigcup_{n \in \mathbb{N}^+} (0, 2 + \frac{1}{n})$ .

(b)  $\bigcap_{n \in \mathbb{N}^+} (0, 2 + \frac{1}{n})$ .

4. (20) Prove that for all non-empty sets  $A$ ,  $B$ , and  $C$ , if  $A \sim B$ , then  $A^C \sim B^C$ .

5. (35) Define a relation  $\rho$  on  $\mathbb{R} \times \mathbb{R}$  so that for all  $\langle a, b \rangle, \langle c, d \rangle \in \mathbb{R} \times \mathbb{R}$ ,

$$\langle a, b \rangle \rho \langle c, d \rangle \text{ iff } a \leq c \text{ and } b \leq d.$$

(a) Prove that  $\rho$  is a partial ordering on  $\mathbb{R} \times \mathbb{R}$ , but not a linear ordering.

(b) For each  $m \in \mathbb{R}^+$ , let  $X_m = \{\langle a, ma \rangle : a \in \mathbb{R}\}$ , then  $X_m \subseteq \mathbb{R} \times \mathbb{R}$ . Prove that for each  $m \in \mathbb{R}^+$ ,  $\rho$  is a linear ordering on  $X_m$ .

6. (15) Prove using mathematical induction that  $2^n > n^2 + 19$ , for all  $n \geq 6$ .

7. (15) Suppose a Function  $f$  is defined so that  $f(0) = 1$  and  $f(n+1) = \sum_{i=0}^n f(i)$ . Prove using strong induction that  $f(n) = 2^{n-1}$ , for  $n \geq 1$ . (Hint:  $\sum_{i=0}^{m-1} 2^i = 2^m - 1$ .)

8. (30) Prove each of the following. ( $c = 2^{\aleph_0}$ ,  $f = c^c$ .)  
(a)  $c = \aleph_0^{\aleph_0} = c^{\aleph_0}$ .

(b)  $f = f^{\aleph_0} = f^c$ .

9. (15) Find the cardinal number of the set of all functions  $g : \mathbb{R} \rightarrow \mathbb{R}$  such that  $g$  is differentiable. Give reasons for your answer. (Hint: Every differentiable function is continuous.)