Name_____

(25) 1.   Show that for all sets A and B,

$$\overline{(A \cup B)} - \overline{(A \cap B)} = (A \cup \bar{B}) - (A \cap \bar{B}).$$

(15) 2. (a) Show that for all sets A and B,
$$\mathscr{P}(A) \cup \mathscr{P}(B) \subseteq \mathscr{P}(A \cup B)$$

(15) (b) Show by constructing a counter example that it is not the case that $\mathscr{P}(A \cup B) \subseteq \mathscr{P}(A) \cup \mathscr{P}(B)$ for all sets A and B.

(20)   3.   If   A =   {(a,b)|(0,1) $\subseteq$ (a,b)},calculate $\bigcap$A   and $\bigcup$A.

(10)   4.   Calculate the cardinality of the set of all circles in the plane with centers on the x-axis. Give reasons for your answer.

5. Suppose g is a real valued function such that for each $x \in \mathbb{R}$,

$$g(x) = \begin{cases} 2x + 1 & \text{if } x \geq 0 \\ x + 1 & \text{if } x < 0. \end{cases}$$

(30)   (a) Show that g is a bijection.

(10)   (b) Calculate $g[(-1,1)]$ and $g^{-1}[(-1,1)]$.

(10)   (c) If h is a real valued function such that $h(x) = x^2$ for each $x \in \mathbb{R}$, calculate $goh(-1)$ and $hog(-1)$.

6. Consider the sequence:
$$x_1 = \sqrt{2}, \quad x_2 = \sqrt{2 + \sqrt{2}}, \quad x_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \text{ etc.}$$

(15)   (a) Give an inductive definition of x_n for $n \geq 1$.

(20)   (b) Prove that $x_n < 2$ for all $n \geq 1$. ($\sqrt{2} = 1.414\ldots$.)

(30) 7. State in your own words the characteristic properties of the following sets.

(a) $\langle \underline{N}, \leq \rangle$

(b) $\langle \underline{Q}, \leq \rangle$

(c) $\langle \underline{R}, \leq \rangle$