MA 387 Final Exam May 6, 1982

Name _____

(15) 1. Show that for all sets X, Y and Z
$(X \cup Y \cup Z) - (Y \cap Z) = (X \cap \overline{Y}) \cup (X \cap \overline{Z}) \cup (Y \cap \overline{Z}) \cup (Z \cap \overline{Y})$

(15) 2. Show that for all sets X, Y and Z
$(X \cap Y) \cup Z = X \cap (Y \cup Z)$ iff $Z \subseteq X$.

(10) 3. If $X = \{a, \{a\}, \{\{a\}\}\}$, calculate $\mathcal{P}(X)$.

(15) 4. If R, S, and T are relations show that
$R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$.

(25) 5. Four relations, ρ_1, ρ_2, ρ_3 and ρ_4 are defined below. The domain of each of ρ_1, ρ_2 and ρ_3 is $\underline{R}$ and the domain of ρ_4 is $[0,1)$.

$u\rho_1 v$ iff $u < v < u + 1$

$u\rho_2 v$ iff $u^2 + v^2 = 1$

$u\rho_3 v$ iff $v = u^2$

$u\rho_4 v$ iff $v = \frac{u}{1 - u}$

(a) Which of the above relations are functions? Give reasons for your answer.

(b) Which of the functions are 1-1 functions? Give reasons for your answer.

(15) 6. Prove that if F is a 1-1 function from A into B and if for all $u \subseteq A$, $G(u) = F[u]$, then G is a 1-1 function from $\mathscr{P}(A)$ into $\mathscr{P}(B)$.

(15) 7. Define R on $\underline{N}$ so that for all $n, m \in \underline{N}$

$$nRm \quad \text{iff} \quad n < m + 2.$$

Show that R is reflexive, but R is not anti-symmetric nor transitive.

(20) 8. $\langle \underline{Q}, \underline{\leq} \rangle$ is a linearly ordered set. $\underline{Z}^+ \subseteq \underline{Q}$ and $\underline{Q}^+ \subseteq Q$.

(a) Does $\underline{Z}^+$ have a $\underline{\leq}$-first element? If so what is it?

(b) Does $\underline{Q}^+$ have a $\underline{\leq}$-first element? If so what is it?

(c) Does $\underline{Z}^+$ have a $\underline{\leq}$-greatest lower bound? If so what is it?

(d) Does $\underline{Q}^+$ have a $\underline{\leq}$-greatest lower bound? If so what is it?

(10) 9. Calculate each of the following:

(a) $\bigcup_{n \in \underline{Z}^+} [1 + \frac{1}{n}, 3 - \frac{1}{n}]$

(b) $\bigcap_{n \in \underline{Z}^+} (1 - \frac{1}{n}, 3 + \frac{1}{n})$

(15) 10. Prove that for all natural numbers $n \geq 6$,

$$2^n > n^2 + 15.$$

(15) 11. Prove that for all natural numbers $n \geq 1$,

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}.$$

(30) 12. For each of the following sets, state whether its cardinal number is $\aleph_0$, c or f and give reasons for your answer.

(a) The set of all subsets of $\underline{N}$.

(b) The set of all finite subsets of $\underline{N}$.

(c) The set of all functions from $\underline{Q}$ to (0,1).

(d) The set of all functions from $\underline{R}$ to {0,1}.