Solution.

MA 387 Exam 1 September 30, 1996

Name____________________ Stud. No.________________

1. (15) Let $A = \{1, 2\}$ and $B = \{2, 3, 4\}$. Find each of the following sets.

(a) $A \cap \overline{B}$. $= \{1\}$

(b) $\mathcal{P}(A)$. $= \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

(c) $A \times B$. $= \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle\}$

2. (10) Prove that for all sets A and B, $A - B = \emptyset$ iff $A \subseteq B$.

$\rightarrow$ Suppose $A - B = \emptyset$ and $x \in A$. If $x \notin B$ ~~because~~ then $x \in A - B$ which is impossible. So $x \in B$
$\therefore A \subseteq B$

$\leftarrow$ Suppose $A \subseteq B$. Then if $x \in A - B$, $x \in A$ and $x \notin B$ which contradicts $A \subseteq B$
$\therefore A - B = \emptyset$

3. (15) Prove that for all sets A, B, and C, $(\overline{A \cup B}) \cup (\overline{B \cup C}) = \overline{B} \cap (\overline{A} \cup \overline{C})$.

$$(\overline{A \cup B}) \cup (\overline{B \cup C}) = (\overline{A} \cap \overline{B}) \cup (\overline{B} \cap \overline{C}) \quad \text{De Morgan}$$

$$= \overline{B} \cap (\overline{A} \cup \overline{C}) \quad \text{Distributive Rule}$$

4. (10) Show that the following statement is NOT a theorem of set theory: For all sets A and B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

Let $A = \{1\}$, $B = \{2\}$

Then $\mathcal{P}(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

$\mathcal{P}(A) = \{\emptyset, \{1\}\}$ $\quad \mathcal{P}(B) = \{\emptyset, \{2\}\}$

$\mathcal{P}(A) \cup \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}\}$.

$\{1, 2\} \in \mathcal{P}(A \cup B)$ but $\{1, 2\} \notin \mathcal{P}(A) \cup \mathcal{P}(B)$.

5. (10) If $f(x) = (7 - \sqrt{x + 1})^{1/3}$, express f as the composition of four functions, none of which is the identity function.

$f_1(x) = x + 1$, $\quad f_2(x) = \sqrt{x}$, $\quad f_3(x) = 7 - x$,

$f_4(x) = x^{1/3}$.

$$f(x) = f_4 \circ f_3 \circ f_2 \circ f_1(x).$$

6. (15) Define a relation ρ on $\mathbb{Z}$ such that $x\rho y$ iff $x - y = 5k$ for some integer k.

(a) Prove that ρ is an equivalence relation.

Reflexive because $x - x = 0 = 5 \cdot 0 \therefore x \rho x$

Symmetric because if $x \rho y$ then $x - y = 5k$
so $y - x = 5(-k) \therefore y \rho x$

Transitive because suppose $x \rho y$ and $y \rho z$.
then $x - y = 5k_1$ and $y - z = 5k_2 \therefore$
$x - z = 5(k_1 + k_2) \therefore x \rho z$.

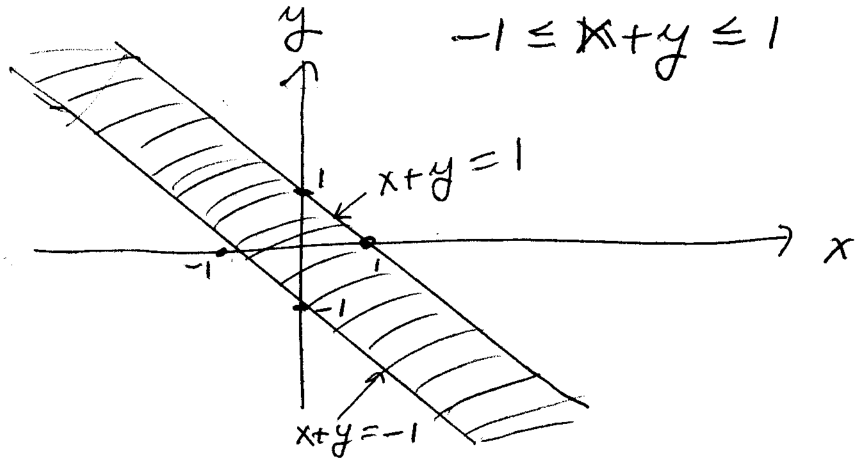
(b) Describe the equivalence classes of ρ.

There are 5 equivalence classes.

the set of all integers which have a remainder of m when divided by 5 for $m = 0, 1, 2, 3, 4$.

7. (10) Sketch a graph of the relation $\rho = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + y| \leq 1\}$.

$-1 \leq x+y \leq 1$



8. (15) Let $f : X \to Y$ be a 1 − 1 function mapping X onto Y and let B and C be any subsets of Y. Prove that $f^{-1}[B - C] = f^{-1}[B] - f^{-1}[C]$.

f^{-1} is a function because f is 1-1.

$x \in f^{-1}[B-C] \leftrightarrow (\exists u \in B-C)(x = f^{-1}(u).)$

$\leftrightarrow (\exists u)(u \in B \text{ and } u \notin C \text{ and } x = f^{-1}(u))$

$\rightarrow x \in f^{-1}[B] \text{ and } x \notin f^{-1}[C]$

$\leftrightarrow x \in f^{-1}[B] - f^{-1}[C].$

$x \in f^{-1}[B] - f^{-1}[C] \leftrightarrow x \in f^{-1}[B] \text{ and } x \notin f^{-1}[C]$

$\leftrightarrow (\exists u \in B)(x = f^{-1}(u)) \text{ and }$

$\neg(\exists v \in C)(x = f^{-1}(v)).$

$\rightarrow (\exists u)(u \in B \;\&\; u \notin C \;\&\; x = f^{-1}(u))$

$\leftrightarrow x \in f^{-1}[B-C].$