# MA 387 Extra Credit Problems

1. Prove that for all $x, y, f, + g$, if $f + g$ are functions then

$$f[x] \cap g[y] = \emptyset \leftrightarrow (g^{-1} \circ f) \cap (x \times y) = \emptyset.$$

Show L.S. $\neq \emptyset \longleftrightarrow$ R.S. $\neq \emptyset$

pf Suppose $u \in$ L.S. then $(\exists a \in x)(\exists b \in y)\ f(a) = g(b)$.

$\therefore \quad g^{-1} \circ f(a) = b \quad \rightarrow \quad \langle a, b \rangle \in$ R.S.

If $\langle a, b \rangle \in$ R.S then $a \in x$, $b \in y$ +

$\langle a, b \rangle \in g^{-1} \circ f \rightarrow (\exists c)(\langle a, c \rangle \in f$ and $\langle c, b \rangle \in g^{-1})$

$\rightarrow (\exists c)(\langle a, c \rangle \in f$ and $\langle b, c \rangle \in g$

$\rightarrow (\exists c)(\ f(a) = g(b) = c\ )$

$\rightarrow c \in$ L.S.

2. Prove that if $f : x \xrightarrow{onto} y$ then $\exists$ a 1-1 function $g$ mapping $\mathcal{P}(y)$ into $\mathcal{P}(x)$.

pf For each $u \subseteq y$ define $F(u) = \{a \in x \mid f(a) \in u\}$

Suppose $v \subseteq y$ + $u \neq v$. Then either

(1) $(\exists a \in u)(a \notin v)$ or (2) $(\exists b \in v)(b \notin u)$.

prove case (1). [ The proof of (2) is similar.

Since $f$ is onto, for each $a \in u$ there is a $c \in x$ such that $a = f(c)$. $\therefore$ $f(c) \in u$ + $f(c) \notin v$.

$\therefore$ $F[u] \neq F[v]$. $\therefore$ $F$ is 1-1.