

**Solutions to the Practice Questions for the Final Exam
MA 153**

1.

$$\frac{\frac{15}{5}}{\frac{1}{2}} = \frac{3}{\frac{1}{2}} = \frac{3}{1} \cdot \frac{2}{1} = 6 \quad \mathbf{D}$$

2.

$$16x^2 - 4y^8 = 4(4x^2 - y^8) = 4[(2x)^2 - (y^4)^2] \quad \mathbf{C}$$

$$= 4(2x + y^4)(2x - y^4)$$

3.

$$\left(\frac{4a^4b^8}{c^{-2}}\right)^{-\frac{1}{2}} = (4a^4b^8c^2)^{-\frac{1}{2}} = \left(\frac{1}{4a^4b^8c^2}\right)^{\frac{1}{2}} \quad \mathbf{A}$$

$$= \frac{\sqrt{1}}{\sqrt{4a^4b^8c^2}} = \frac{1}{2a^2b^4c}$$

4.

$$\frac{3x}{3x+1} - \frac{x}{x-2} = \frac{3x(x-2)}{(3x+1)(x-2)} - \frac{x(3x+1)}{(3x+1)(x-2)} \quad \mathbf{C}$$

$$= \frac{\cancel{3x^2} - 6x - \cancel{3x^2} - x}{(3x+1)(x-2)} = \frac{-7x}{(3x+1)(x-2)}$$

5.

$$\frac{x-2}{(x+1)(x-3)} \div \frac{(x-2)(x+1)}{(x+3)(x-3)} \quad \mathbf{B}$$

$$= \frac{\cancel{x-2}}{(x+1)(x-3)} \cdot \frac{(x+3)(x-3)}{\cancel{(x-2)}(x+1)} = \frac{x+3}{(x+1)^2}$$

6. let $t =$ time second person working alone

$$\frac{1}{6} + \frac{1}{t} = \frac{1}{4} \quad \text{or} \quad \frac{1}{6}(4) + \frac{1}{t}(4) = 1$$

$$12t \left[\frac{1}{6} + \frac{1}{t} \right] = \left[\frac{1}{4} \right] 12t \quad 3t \left[\frac{2}{3} + \frac{4}{t} \right] = [1] 3t \quad \mathbf{E}$$

$$2t + 12 = 3t \quad 2t + 12 = 3t$$

$$t = 12 \quad t = 12$$

$$7. \quad \frac{xy^{-1}}{(x+y)^{-1}} = \frac{\frac{x}{y}}{\frac{1}{(x+y)}} = \frac{x}{y} \cdot \frac{(x+y)}{1} = \frac{x(x+y)}{y} \quad \mathbf{A}$$

$$8. \quad \frac{\sqrt{3}}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} \quad \mathbf{C}$$

$$= \frac{2\sqrt{3} - (\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = \frac{2\sqrt{3} - 3}{4 - 3} = 2\sqrt{3} - 3$$

9. let $x =$ first positive integer ($x < y$)

$$\begin{cases} y = x + 1 \\ y^2 - x^2 = 145 \end{cases} \Rightarrow \text{substitute eq. (1) into eq.(2) for } y:$$

$$(x+1)^2 - x^2 = 145$$

$$x^2 + 2x + 1 - x^2 = 145 \quad \mathbf{B}$$

$$2x + 1 = 145$$

$$2x = 144$$

$$x = 72$$

10.

$$A = P(1 + rt)$$

$$A = P + Prt$$

$$A - P = Prt$$

E

$$\frac{A - P}{Pr} = t$$

11. let $t = \#$ hours truck has been traveling

$$40t = 55(t - 1)$$

$$40t = 55t - 55$$

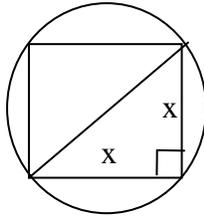
$$55 = 15t$$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t - 1)

$$t = \frac{55}{15} = \frac{11}{3} \text{ hours, so distance is } 40\left(\frac{11}{3}\right) = \frac{440}{3} \text{ miles}$$

A

12.



Let $A =$ area of circle

$$\text{Area of circle} \Rightarrow A(r) = \pi r^2$$

$$\text{Diameter } (d) \text{ of circle} \Rightarrow x^2 + x^2 = d^2$$

$$2x^2 = d^2$$

$$d = \pm\sqrt{2x^2}$$

$$d = x\sqrt{2}$$

A

$$\text{Radius } (r) \text{ of circle} \Rightarrow r = \frac{x\sqrt{2}}{2}$$

$$\text{So, } A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$$

$$= \frac{\pi x^2}{2} \text{ or } \frac{\pi}{2} x^2$$

13.

$$\frac{4}{2p-3} + \frac{10}{4p^2-9} = \frac{1}{2p+3} \quad \left(\text{Domain : } p \neq \pm \frac{3}{2} \right)$$

$$(2p+3)(2p-3) \left[\frac{4}{2p-3} + \frac{10}{(2p+3)(2p-3)} \right] = \left[\frac{1}{2p+3} \right] (2p+3)(2p-3)$$

$$4(2p+3) + 10 = 2p-3$$

D

$$8p + 12 + 10 = 2p - 3$$

$$6p = -25$$

$$p = -\frac{25}{6}$$

14.

let $x = \#$ ml of the 50% solution

let $y =$ total # of ml

$$\begin{cases} x + 40 = y \\ x(.50) + 40(.20) = y(.25) \end{cases}$$

B

$$x(.50) + 8 = (x + 40)(.25)$$

$$.50x + 8 = .25x + 10$$

$$.25x = 2$$

$$x = 8 \text{ ml}$$

15.

$$x = \sqrt{14 + 5x}$$

Check :

$$(x)^2 = (\sqrt{14 + 5x})^2$$

If $x = 7$:

$$x^2 = 14 + 5x$$

$$7 = \sqrt{14 + 35}$$

$$x^2 - 5x - 14 = 0$$

$$7 = \sqrt{49} \Rightarrow \text{yes}$$

E [$x = 7$]

$$(x-7)(x+2) = 0$$

If $x = -2$:

$$x = 7, \quad x = \cancel{-2}$$

$$-2 = \sqrt{14 - 10}$$

$$-2 = \sqrt{4} \Rightarrow \text{no}$$

16

$$m^4 - m^2 - 6 = 0$$

$$(m^2 - 3)(m^2 + 2) = 0$$

$$m^2 - 3 = 0, \quad m^2 + 2 = 0$$

$$m^2 = 3, \quad m^2 = -2$$

$$m = \pm\sqrt{3}, \quad m = \pm\sqrt{-2}$$

$$m = \pm i\sqrt{2}$$

D

17.

$$3x - 2 > 6x + 1$$

$$-3x > 3$$

$$x < -1$$

$$(-\infty, -1)$$

A

18.

$$|6 - 2x| \leq 3$$

$$-3 \leq 6 - 2x \leq 3$$

$$-9 \leq -2x \leq -3$$

$$\frac{9}{2} \geq x \geq \frac{3}{2}$$

$$\frac{3}{2} \leq x \leq \frac{9}{2}$$

C

19.

$$2x^2 - 4x + k = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(2)(k)}}{2(2)} = \frac{4 \pm \sqrt{16 - 8k}}{4}$$

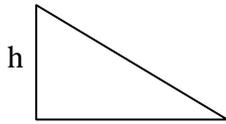
$$\text{Need: } 16 - 8k \geq 0$$

$$-8k \geq -16$$

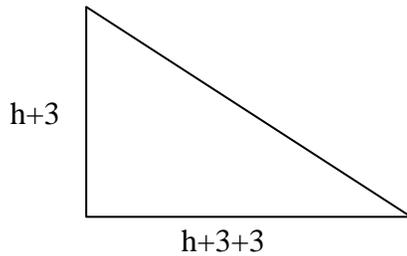
$$k \leq 2$$

D

20.



New: $h+3$



Let h = height of original triangle
then base of original triangle = $h+3$

Area of new triangle = 14 in^2

$$\frac{1}{2}(h+3)(h+6) = 14$$

$$(h+3)(h+6) = 28$$

$$h^2 + 3h + 6h + 18 = 28$$

$$h^2 + 9h - 10 = 0$$

A

$$(h+10)(h-1) = 0$$

$$h = -10, h = 1$$

Original height = 1 in.

Original base = $1 + 3 = 4 \text{ in.}$

21.

$$\begin{cases} 2x^2 + y^2 = 1 \\ x - y = 1 \Rightarrow y = x - 1 \end{cases}$$

$$2x^2 + (x-1)^2 = 1$$

$$2x^2 + x^2 - 2x + 1 = 1$$

B

$$3x^2 - 2x = 0$$

$$x(3x-2) = 0$$

$$x = 0, x = \frac{2}{3}$$

22.

A (1,-2), Midpoint M (2,3), B (x,y)

$$\text{Midpoint} \Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Midpoint of } \overline{AB} \Rightarrow \left(\frac{1+x}{2}, \frac{-2+y}{2} \right) \Rightarrow (2,3)$$

$$\frac{1+x}{2} = 2, \quad \frac{-2+y}{2} = 3 \quad \mathbf{C}$$

$$1+x = 4, \quad -2+y = 6$$

$$x = 3, \quad y = 8$$

so B (3,8)

23.

$$\text{slope of line} \Rightarrow m = -\frac{1}{3}$$

D

$$\text{slope of line perpendicular} \Rightarrow m = 3$$

24.

$$m = \frac{xyk}{z} \quad 3 = \frac{(4)(2)k}{6}$$

$$18 = 8k \quad \mathbf{B}$$

$$k = \frac{18}{8} = \frac{9}{4}$$

25.

$$2x - 3y = 7$$

$$-3y = -2x + 7$$

$$y = \frac{2}{3}x - \frac{7}{3}$$

$$\text{slope } m = \frac{2}{3}$$

$$\text{slope of parallel line } m = \frac{2}{3}$$

$$\text{point is } (4,-1); m = \frac{2}{3}$$

$$y = mx + b$$

$$-1 = \left(\frac{2}{3} \right)(4) + b \quad \mathbf{C}$$

$$-1 = \frac{8}{3} + b$$

$$b = -\frac{11}{3} \quad \text{so } y = \frac{2}{3}x - \frac{11}{3}$$

26.

$$\text{Center} \Rightarrow (0,2) \quad (x-h)^2 + (y-k)^2 = r^2$$

$$\text{radius} = 2 \quad (x-0)^2 + (y-2)^2 = 2^2$$

$$x^2 + (y-2)^2 = 4$$

B

$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + y^2 - 4y = 0$$

27.

$$f(x) = 1 - \sqrt{x}, \quad g(x) = \frac{1}{x}$$

D

$$(g \circ f)(x) = g[f(x)] = g(1 - \sqrt{x}) = \frac{1}{1 - \sqrt{x}}$$

28.

$$f(x) = \frac{x}{x^2 + 1}$$

D

$$\frac{1}{f(3)} = \frac{1}{\frac{3}{(3)^2 + 1}} = \frac{1}{\frac{3}{10}} = \frac{10}{3}$$

29.

$$\text{Vertex} \Rightarrow V(0,2) \quad y = a(x-h)^2 + k$$

$$\text{point on parabola} \Rightarrow (1,0) \quad y = a(x-0)^2 + 2$$

$$y = ax^2 + 2$$

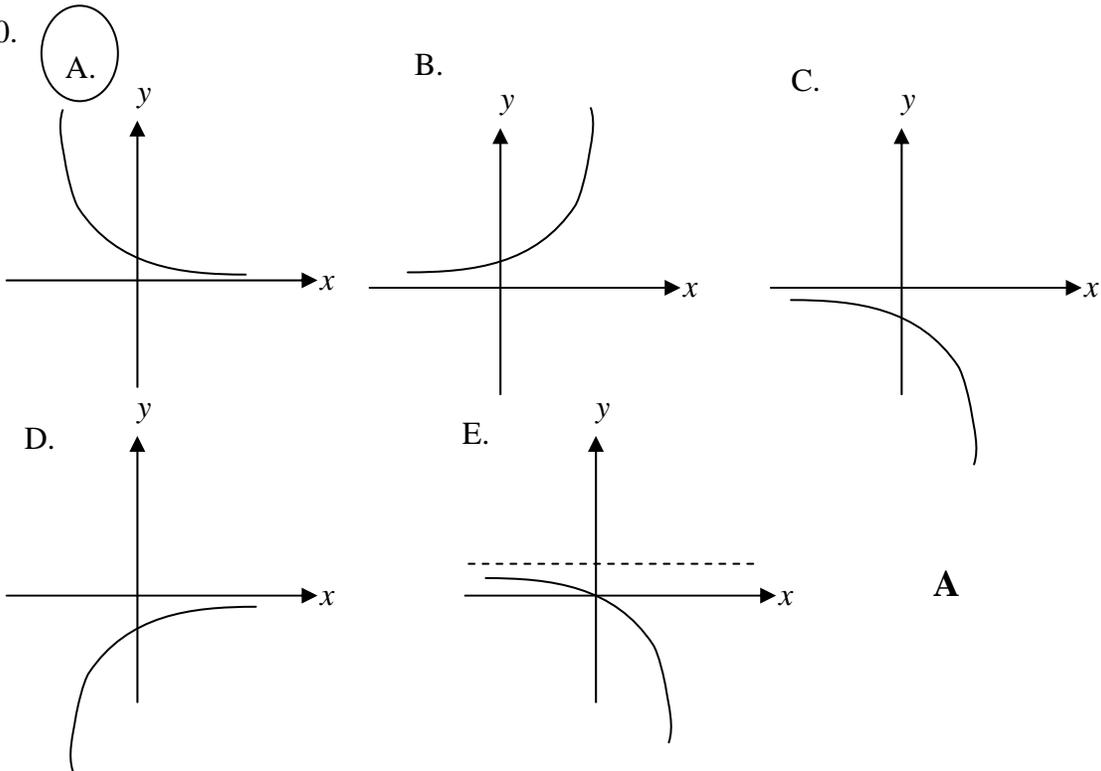
B

$$0 = a(1)^2 + 2$$

$$a = -2$$

$$y = -2x^2 + 2$$

30.



31.

$$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b (y^3 y^2) - \log_b y^4$$

$$= \log_b y^5 - \log_b y^4 = \log_b \left(\frac{y^5}{y^4} \right) = \log_b y$$

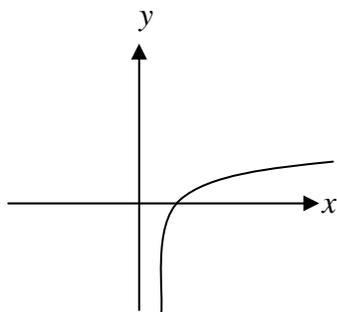
B

32.

$$f(x) = \log_a x \text{ if } a > 1$$

example : if $a = 2$, then $f(x) = \log_2 x$,

Graph of $y = \log_2 x \Rightarrow 2^y = x$



f is increasing, f does not have a as an x -intercept (the x -int. is $(1,0)$), f does not have a y -intercept, the domain of f is $(0, \infty)$.

D

33.

$$\log\left(\frac{432}{(\sqrt{.095})(\sqrt[3]{72.1})}\right) = \log\left(\frac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$$

$$= \log 432 - \left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$$

B

$$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}}\right)$$

$$= \log 432 - \frac{1}{2}\log .095 - \frac{1}{3}\log 72.1$$

34.

$$3^{x-5} = 4$$

$$\log 3^{x-5} = \log 4$$

$$(x-5)\log 3 = \log 4$$

C

$$x-5 = \frac{\log 4}{\log 3}$$

$$x = \frac{\log 4}{\log 3} + 5$$

35.

$$\log_3 \sqrt{2x+3} = 2$$

$$3^2 = \sqrt{2x+3}$$

$$\sqrt{2x+3} = 9$$

$$(\sqrt{2x+3})^2 = (9)^2$$

$$\text{Check: } \sqrt{2(39)+3} = 9$$

C

$$2x+3 = 81$$

$$9 = 9$$

$$2x = 78$$

$$\text{Check: } \log_3 \sqrt{2(39)+3} = 2$$

$$x = 39$$

$$3^2 = \sqrt{81}$$

36.

$$\log_3 m = 8 \qquad \log_3 \left(\frac{\sqrt{mn}}{p^3} \right) = \log_3 (mn)^{\frac{1}{2}} - \log_3 p^3$$

$$\log_3 n = 10 \quad \Rightarrow \quad = \log_3 \left(m^{\frac{1}{2}} n^{\frac{1}{2}} \right) - \log_3 p^3$$

$$\begin{aligned} \log_3 p = 6 \qquad &= \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3 && \mathbf{A} \\ &= \frac{1}{2} \log_3 m + \frac{1}{2} \log_3 n - 3 \log_3 p \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}(8) + \frac{1}{2}(10) - 3(6) \\ &= 4 + 5 - 18 = -9 \end{aligned}$$

37.

$$y = 2 + 2^x$$

$$\text{When } x = 0, \quad y = 2 + 2^0 \qquad \mathbf{D}$$

$$y = 2 + 1 = 3$$

38.

$$y = x^2(x-1)(x+1)^2$$

$$\text{x - intercepts: } x^2(x-1)(x+1)^2 = 0$$

$$x = 0, \quad x = 1, \quad x = -1$$

A

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
x^2	+	+	+	+
$x - 1$	-	-	-	+
$(x + 1)^2$	+	+	+	+
Result	-	-	-	+
	below	below	below	above
	x - axis	x - axis	x - axis	x - axis

39.

$$\begin{cases} 2x - 3y = 4 \\ -4x + 6y = 3 \end{cases} \Rightarrow 2E_1 \begin{cases} 4x - 6y = 8 \\ -4x + 6y = 3 \end{cases} \Rightarrow E_1 + E_2$$

Answer is **C** because:

$$\begin{cases} 4x - 6y = 8 \\ 0 = 11 \end{cases} \Rightarrow \text{No Solution}$$

40.

$$\begin{cases} x + 4y = 3 \\ 2x - 6y = 8 \end{cases} \Rightarrow -2E_1 \begin{cases} -2x - 8y = -6 \\ 2x - 6y = 8 \end{cases} \Rightarrow E_1 + E_2$$

$$\begin{cases} -2x - 8y = -6 \\ -14y = 2 \end{cases} \Rightarrow y = -\frac{2}{14} = -\frac{1}{7} \quad \mathbf{E}$$

$$x + 4\left(-\frac{1}{7}\right) = 3$$

$$x = \frac{25}{7} \quad \text{so} \left(\frac{25}{7}, -\frac{1}{7}\right)$$

41.

let t = number of years after 1980 and let V = value
 t is the independent variable and V is the dependent variable

points on line $\Rightarrow (1, 54)$ and $(3, 62)$

$$\text{slope} \Rightarrow m = \frac{62 - 54}{3 - 1} = \frac{8}{2} = 4$$

$$V - V_1 = m(t - t_1) \quad \mathbf{A}$$

$$V - 54 = 4(t - 1)$$

$$V - 54 = 4t - 4$$

$$V = 4t + 50$$

42.

$$f(x) = x^2 - 2x + 4$$

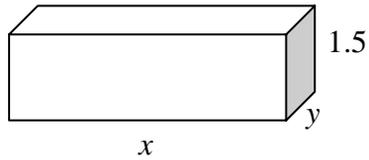
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + 4 - \cancel{x^2} + \cancel{2x} - 4}{h} = \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = 2x + h - 2$$

A

43.



$$\text{Volume} = 6 \text{ ft.}^3$$

$$xy(1.5) = 6$$

$$y = \frac{6}{1.5x}$$

B

$$y = \frac{4}{x}$$

44.

$$\log_x 2 = 5$$

$$x^5 = 2$$

$$(x^5)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$$

D

$$x = \sqrt[5]{2}$$

$$x \approx 1.1487$$