1. The distance between the points $(1, 1)$ and $(3, 0)$ is

A. $\sqrt{5}$
B. 2
C. $\sqrt{3}$
D. $\sqrt{2}$
E. 1

$$\text{distance} = \sqrt{(1-3)^2 + (1-0)^2}$$
$$= \sqrt{(-2)^2 + (1)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$

2. The domain of the function $f(x) = \dfrac{x+1}{\sqrt{|2x+3|-1}}$ is

A. $(-\infty, -2) \cup (-1, \infty)$
B. $(-1, \infty)$
C. $(-\infty, 0) \cup (1, \infty)$
D. $(-\infty, \infty)$
E. There is no solution

$\therefore |2x+3| - 1 \geq 0$

$\rightarrow |2x+3| \geq 1$

$\rightarrow 2x+3 \geq 1$ or $2x+3 \leq -1$

$\rightarrow 2x \geq -2$ or $2x \leq -4$

$\rightarrow x \geq -1$ or $x \leq -2$

$\rightarrow (-\infty, -2) \cup (-1, \infty)$

3. Let L be a straight line through $(0, 1)$ and parallel to the line $2x + 3y + 1 = 0$. Then an equation for L is

A. $3x + 2y + 3 = 0$

B. $y = -\frac{2}{3}\, x + 1$

C. $y = \frac{3}{2}\, x + 1$

D. $5x - y + 1 = 0$

E. $x + 5y = 0$

line $2x + 3y + 1 = 0$

$\rightarrow 3y = -2x - 1$

$\rightarrow y = -\frac{2}{3}x - \frac{1}{3}$

$\rightarrow$ slope of line is $\frac{-2}{3}$

line L has slope $-\frac{2}{3}$ and contains pt. $(0,1)$

line L has an equation $y - 1 = -\frac{2}{3}(x-0)$

$\rightarrow y = -\frac{2}{3} + 1$

4. $\tan(\sin^{-1} x) =$

A. $\cos^{-1} x$

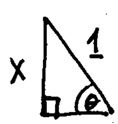
B. $\sqrt{1 + \tan^2 x}$

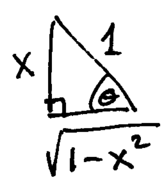
C. $\sqrt{1 + x^2}$

D. $\dfrac{\sqrt{1 + x^2}}{x}$

E. $\dfrac{x}{\sqrt{1 - x^2}}$

Let $\theta = \sin^{-1} x$. Then $\sin\theta = x = \frac{x}{1}$,

and 

and 

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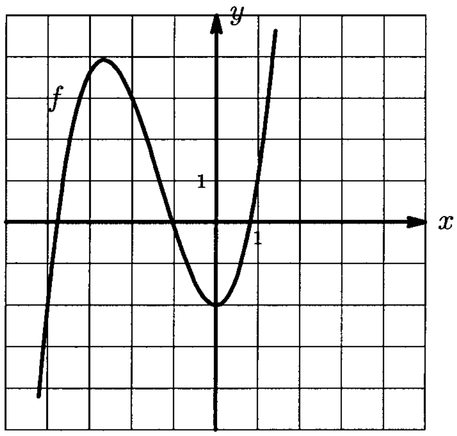
Thus $\tan(\sin^{-1} x) = \tan\theta = \dfrac{x}{\sqrt{1-x^2}}$

5. The graph of f is given. Then $f(f(0)) =$



$f(0) = -2.$

$f(f(0)) = f(-2) = 3$

A. -2

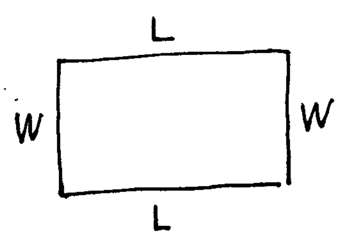
B. 0

C. 3

D. -1

E. undefined

6. A rectangle has area 25 (square inches) and one of its sides has length L (inches). Express the perimeter P (in inches) as a function of L.



given: $25 = LW$

A. $P = 2L + \dfrac{10}{L}$

B. $P = 2L + \dfrac{50}{L}$

C. $P = L + \dfrac{50}{L}$

D. $P = L + 50L^2$

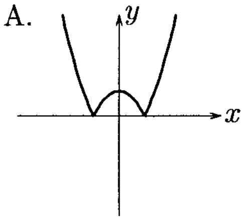
E. $P = 2L - 25L^2$

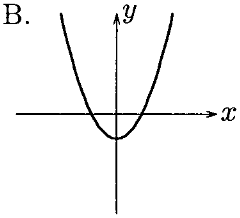
$P = 2L + 2W.$

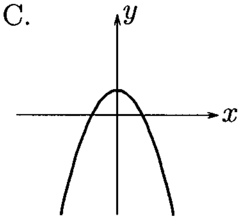
$25 = LW \rightarrow W = \dfrac{25}{L}$

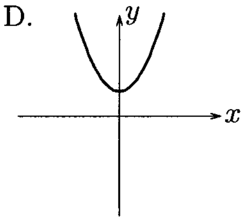
Then $P = 2L + 2\left(\dfrac{25}{L}\right) = 2L + \dfrac{50}{L}$

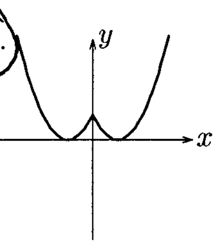
7. The graph of $y = (1 - |x|)^2$ looks like

A.


B.


C.


D.


E.


$y = (1 - |x|)^2$

$= \begin{cases} (1-x)^2 & \text{if } x \geq 0 \\ (1+x)^2 & \text{if } x < 0. \end{cases}$

* see bottom of page for graphs

8. In a certain colony of bacteria population triples every 7 hours. Suppose initially there are 1,000 bacteria. After 10 hours the population is

$P(0) = 1{,}000 \quad = 3^0 \cdot 1{,}000 = 3^{0/7} \cdot 1{,}000$

$P(7) = 3 \cdot 1{,}000 \quad = 3^1 \cdot 1{,}000 = 3^{7/7} \cdot 1{,}000$

$P(14) = 3 \cdot 3 \cdot 1{,}000 \quad = 3^2 \cdot 1{,}000 = 3^{14/7} \cdot 1{,}000$

$P(21) = 3 \cdot 3 \cdot 3 \cdot 1{,}000 \quad = 3^3 \cdot 1{,}000 = \underbrace{3^{21/7} \cdot 1{,}000}$

$\vdots$

$P(t) = 3^{t/7} \cdot 1{,}000$

Thus, $P(10) = 3^{10/7} \cdot 1{,}000$

A. $1{,}000 \cdot 3^7$

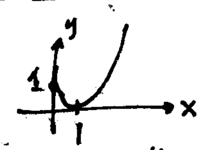
B. $1{,}000 \cdot 3^{10}$

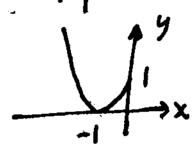
C. $1{,}000 \cdot 3^{70}$

D. $1{,}000 \cdot 3^{10/7}$

E. $1{,}000 \cdot 3^{1.7}$

* #7 (continued)

$x \geq 0 \rightarrow y = (1-x)^2 = (x-1)^2$ and graph is


$x < 0 \rightarrow y = (1+x)^2 = (x+1)^2$ and graph is


Putting 2 graphs together gives


9. The solution(s) of the equation $\ln(\ln x) = 0$ is (are)

$\ln(\ln x) = 0$

$\rightarrow (\ln x) = e^0$

$\rightarrow \ln x = 1$

$\rightarrow x = e^1$

$\rightarrow x = e$

A. $x = 1$

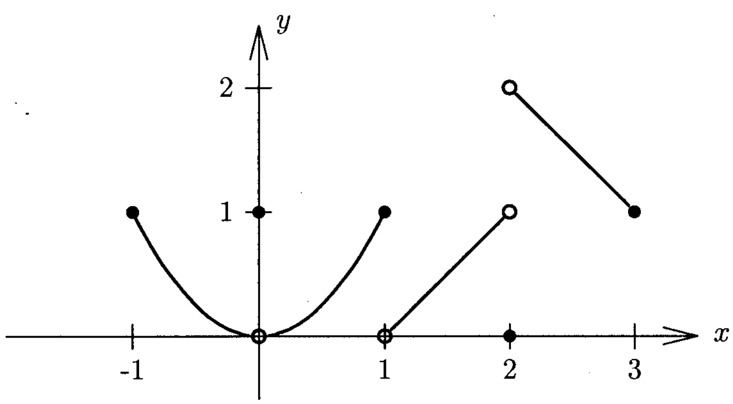
B. $x = e$

C. $x = 1$ and e

D. $x = e^2$

E. The equation has no solution

10. Given the following graph of $f(x)$, which statement is true?



A. $\lim_{x\to 2^+} f(x) = 1$

B. $\lim_{x\to 2^+} f(x) = 0$

C. $\lim_{x\to 2^-} f(x)$ does not exist

D. $\lim_{x\to 2} f(x) = 1$ and 0

E. $\lim_{x\to 2^-} f(x) = 1$

A. limit point as $x \to 2^+$ is $(2, 2)$ so $\lim_{x\to 2^+} f(x) = 2$. A is FALSE.

B. FALSE. (see A.)

C. limit point as $x \to 2^-$ is $(2, 1)$ so $\lim_{x\to 2^-} f(x) = 1$. C is FALSE.

D. FALSE. If a limit exists then it has only one value.

E. TRUE (see C.)

11. If $\lim_{x\to a} f(x) = 4$, $\lim_{x\to a} g(x) = -3$, and $\lim_{x\to a} h(x) = 0$, it follows that $\lim_{x\to a} \frac{f(x)g(x)}{h(x)^2}$ is

$$\lim_{x\to a} \frac{f(x)\cdot g(x)}{h(x)^2} = \frac{4\cdot -3}{0^+} = \frac{-12}{0^+} = -\infty$$

A. -12

B. 0

C. ∞

D. $-\infty$

E. impossible to determine

The numerator is approaching -12

The denominator is approaching 0 through positive values.

Hence the ratio $\frac{f(x)g(x)}{h(x)^2}$ is negative and becoming infinitely large.

Thus the limit is approaching $-\infty$.

12. $\lim_{x\to 1} e^{x^2-x} =$ $e^{1-1} = e^0 = 1$.

A. e

B. e^{x^2-x}

C. 1

D. 0

E. ∞

13. If $\lim_{t\to 4} s(t) = -2$, and $\lim_{t\to 4}[3r(t) + 2s(t)] = -1$, then $\lim_{t\to 4} r(t) =$

A. 1

B. -1

C. -2

D. 0

E. cannot be determined.

$$\lim_{t\to 4}\left(3\,r(t) + 2\,s(t)\right) = -1$$

$$\to 3\cdot\lim_{t\to 4} r(t) + 2\cdot\lim_{t\to 4} s(t) = -1$$

$$\to 3\cdot\lim_{t\to 4} r(t) + 2\cdot(-2) = -1$$

$$\to 3\lim_{t\to 4} r(t) = 3$$

$$\to \lim_{t\to 4} r(t) = 1.$$

14. The graph of $y = f(x)$ is reflected about the y–axis, then translated down 4 units and to the right 3 units, and finally compressed horizontally by a factor of 2. The resulting graph has equation

A. $y = f\left(-\left(\frac{1}{2}x - 3\right)\right) - 4$

B. $y = f\left(-\left(\frac{1}{2}x - 3\right)\right) + 4$

C. $y = f(-(2x-3)) + 4$

D. $y = f(-(2x-3)) - 4$

E. $y = f(-(2x+3)) - 4$

reflect about y-axis $\to y = f(-x)$

translate down 4 units $\to y = f(-x) - 4$

translate to right 3 units

$\to y = f(-(x-3)) - 4$

compress horizontally by factor of 2,
which is same as scale horizontally by factor of $\frac{1}{2}$

$$\to y = f\left(-\left(\frac{x}{\frac{1}{2}} - 3\right)\right) - 4$$

$$= f(-(2x-3)) - 4.$$