MA 161/161E EXAM 2 Spring 2001 February 27, 2001

1. Let $f$ be a function defined on $(-\infty, \infty)$. Which is (are) true?

( I.) $f$ is continuous at $a$ if $\lim_{x \to a} f(x) = f(a)$.

( II.) $f$ is continuous at $a$ if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$.

( III.) If $f$ is differentiable, then $f$ is continuous.

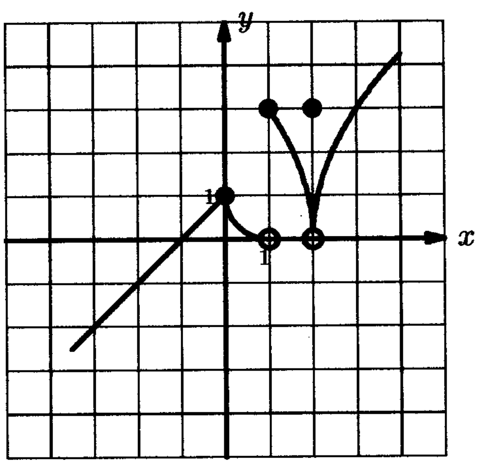
A. Only I.
B. Only II.
C. Only I and III.
D. Only I and II.
E. Only II and III.

I is true. This is the definition of continuity at a point.

II is false. Equality of the right- and left-hand limits only imply existance of $\lim_{x \to a} f(x)$. That limit may not equal $f(a)$.

III is true. See Theorem 4, page 169 in Stewart text.

2. The numbers where the function sketched below is discontinuous are




A. 0, 1, 2
B. 1, 2
C. 1, 3
D. 2, 3
E. 0, 2

The function is discontinuous at $x=1$ because $\lim_{x \to 1} f(x)$ does not exist, and at $x=2$ because $\lim_{x \to 2} f(x) = 0$ but $f(2) = 3$.

3. The horizontal asymptote of the function $f(x) = \dfrac{8x + 6x^3}{12x^2 - 2x^3}$ is the line

A. $y = -3$

B. $y = \dfrac{2}{3}$

C. $y = \dfrac{1}{2}$

D. $y = 0$

E. No horizontal asymptote exists.

$$\lim_{x \to \pm\infty} \frac{8x + 6x^3}{12x^2 - 2x^3} = \frac{\pm\infty}{\pm\infty}$$

$$= \lim_{x \to \pm\infty} \frac{(8x + 6x^3)\left(\frac{1}{x^3}\right)}{(12x^2 - 2x^3)\left(\frac{1}{x^3}\right)}$$

$$= \lim_{x \to \pm\infty} \frac{\frac{8}{x^2} + 6}{\frac{12}{x} - 2} = \frac{0+6}{0-2} = -3.$$

$y = -3$ is the horizontal asymptote.

4. $\displaystyle\lim_{x\to\infty} \left(\sqrt{2x^2+4x} - \sqrt{2x^2-4x}\right) =$ $\infty - \infty$

A. 0

B. $\infty$

C. 4

D. $\sqrt{2}$

E. $2\sqrt{2}$

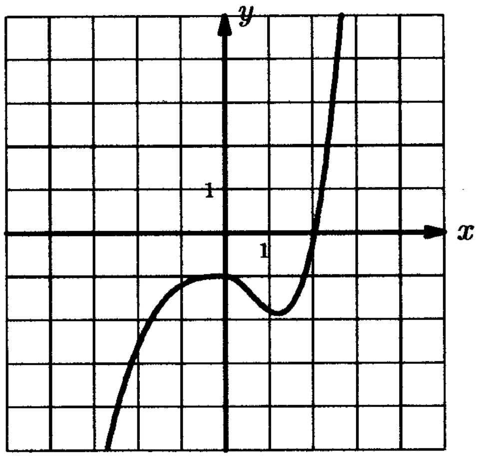
$$= \lim_{x \to \infty} \left(\sqrt{2x^2+4x} - \sqrt{2x^2-4x}\right)\left(\frac{\sqrt{2x^2+4x} + \sqrt{2x^2-4x}}{\sqrt{2x^2+4x} + \sqrt{2x^2-4x}}\right)$$

$$= \lim_{x \to \infty} \frac{(2x^2+4x) - (2x^2-4x)}{\sqrt{2x^2+4x} + \sqrt{2x^2-4x}}$$

$$= \lim_{x \to \infty} \frac{8x}{\sqrt{2x^2+4x} + \sqrt{2x^2-4x}} = \lim_{x \to \infty} \frac{8x}{\sqrt{x^2\left(2+\frac{4}{x}\right)} + \sqrt{x^2\left(2-\frac{4}{x}\right)}}$$

$$= \lim_{x \to \infty} \frac{8x}{\sqrt{x^2}\sqrt{2+\frac{4}{x}} + \sqrt{x^2}\sqrt{2-\frac{4}{x}}} = \lim_{x \to \infty} \frac{8}{\sqrt{2+\frac{4}{x}} + \sqrt{2-\frac{4}{x}}} = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$$

5. If $f$ is given by this graph, which of the statements I, II and III is (are) true?




I. $f'(-2) < 0$

II. $f'(0) = 0$

III. $f'(2) > 0$

A. Only I and II.

B. Only II and III.

C. Only II.

D. All are true.

E. None are true.

I is False. The tangent line at $x=-2$ has positive slope.
II is True. The tangent line at $x=0$ is horizontal
III is True. The Tangent line at $x=2$ has positive slope.

6. If the tangent line to $y = f(x)$ at $(1, 2)$ passes through the point $(3, 2)$, then

A. $f(1) = 2, f'(3) = 2$

B. $f(2) = 1, f'(3) = 2$

C. $f(1) = 2, f'(1) = 0$

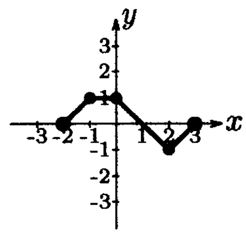
D. $f(1) = 3, f'(1) = 2$

E. $f$ is not differentiable at $(1, 2)$.

Tangent line passes through $(1,2)$ and $(3,2)$ and hence has slope $\frac{2-2}{3-1} = \frac{0}{2} = 0$.
Thus $f'(x) = 0$ for all $x$.

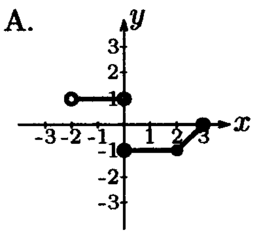
Since point of tangency is $(1,2)$, then $f(1) = 2$.

7. Given the following graph of a function $f$, the graph of $f'$ would look most like

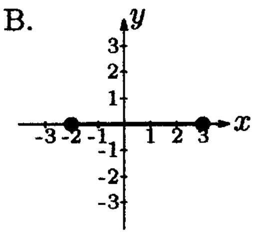


$$\Rightarrow f'(x) = \begin{cases} 1, & -2 < x < -1 \\ 0, & -1 < x < 0 \\ -1, & 0 < x < 2 \\ 1, & 2 < x < 3 \end{cases}$$

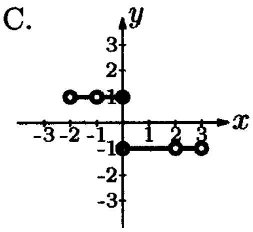
A.



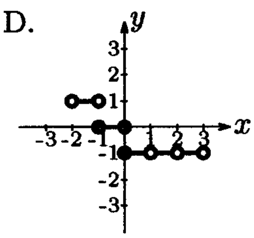
B.



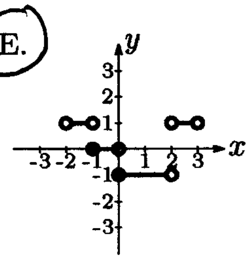
C.



D.



E.



8. Let $y = e^{x^2+1}$. Then $\dfrac{dy}{dx}$ at $x = 2$ is

$$\frac{dy}{dx} = \left(e^{x^2+1}\right)(2x)$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \left(e^{2^2+1}\right)(2.2) = 4e^5$$

A. $e^2$

B. $e^4$

C. $e^5$

D. $4e^5$

E. $4e^4$

9. Let $f(x) = x^3 \cos\left(\frac{\pi x}{2}\right)$. Then $f'(1) =$

$$f'(x) = (3x^2)\left(\cos\left(\frac{\pi x}{2}\right)\right) + (x^3)\left(-\sin\left(\frac{\pi x}{2}\right)\right)\left(\frac{\pi}{2}\right)$$

$$f'(1) = (3)\left(\cos\frac{\pi}{2}\right) + (1)\left(-\sin\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)$$

$$= (3)(0) + (1)(-1)\left(\frac{\pi}{2}\right)$$

$$= -\frac{\pi}{2}$$

A. $-\frac{\pi}{2}$
B. $3 - \frac{\pi}{2}$
C. $\frac{3\pi}{2}$
D. $-1$
E. $\pi$

10. Given $f(x) = \frac{x-1}{x+1}$, then $f'(x) =$

$$f'(x) = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2}$$

A. $\frac{2}{(x+1)^2}$
B. $\frac{2x}{(x+1)^2}$
C. $\frac{2}{x+1}$
D. $\frac{-2x}{x+1}$
E. $\frac{-2}{(x+1)^2}$

11. If $f(x) = \dfrac{x^2}{g(x)}$, where $g(2) = \sqrt{3}$ and $f'(2) = 2$, then $g'(2)$ is

A. $\dfrac{3-\sqrt{3}}{2}$

B. $\sqrt{3} - \dfrac{3}{2}$

C. $\dfrac{3\sqrt{3}}{2}$

D. $\dfrac{-3\sqrt{3}}{2}$

E. cannot be determined

$$f'(x) = \frac{(2x)(g(x)) - (x^2)(g'(x))}{(g(x))^2}$$

$$\rightarrow f'(2) = \frac{4g(2) - 4g'(2)}{(g(2))^2}$$

$$\rightarrow 2 = \frac{4\sqrt{3} - 4g'(2)}{3} \rightarrow 6 = 4\sqrt{3} - 4g'(2)$$

$$\rightarrow g'(2) = \frac{6-4\sqrt{3}}{-4} = -\frac{3}{2} + \sqrt{3}$$

12. Given $f(t) = \sqrt[3]{t^2} + 2\sqrt{t^3}$, $f'(t) =$

A. $\dfrac{3}{2\sqrt[3]{t^2}} + \sqrt{t}$

B. $\dfrac{2}{3\sqrt[3]{t^2}} + \sqrt{t}$

C. $\dfrac{2}{3\sqrt[3]{t}} + 2\sqrt{t}$

D. $\dfrac{2}{3\sqrt[3]{t}} + 3\sqrt{t}$

E. $\dfrac{3}{2\sqrt[3]{t}} + 3\sqrt{t}$

$$f(t) = (t^2)^{1/3} - 2(t^3)^{1/2}$$

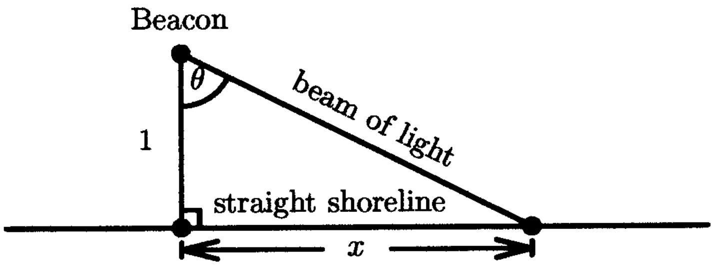
$$= t^{2/3} - 2t^{3/2}$$

$$f'(t) = \frac{2}{3}t^{-1/3} - 2 \cdot \frac{3}{2} t^{1/2}$$

$$= \frac{2}{3t^{1/3}} - 3t^{1/2}$$

$$= \frac{2}{3\sqrt[3]{t}} - 3\sqrt{t}$$

13. A rotating beacon of light is located 1 km from a straight shoreline. See the figure below. What is the rate of change of $x$ with repect to $\theta$ (in km/rad) at $x = 2$ km?




A. 2

B. $\sqrt{5}$

C. $\frac{\sqrt{5}}{2}$

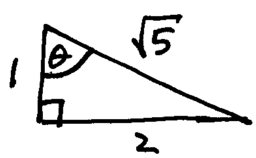
D. 1

E. 5

$$\tan\theta = x$$

$$\rightarrow \quad \sec^2\theta = \frac{dx}{d\theta}$$

$x = 2 \rightarrow$




$$\rightarrow \quad \frac{dx}{d\theta} = \sec^2\theta = \left(\frac{\sqrt{5}}{1}\right)^2 = 5$$

14. If $y = \left(\tan(x^4 + x)\right)^3$, then $\frac{dy}{dx}$ at $x = 1$ is

A. $3(\tan^2 2)(\sec^2 2)$

B. $5(\tan^2 2)(\sec^2 2)$

C. $10(\tan^2 2)(\sec^2 2)$

D. $15(\tan^2 2)(\sec^2 2)$

E. $20(\tan^2 2)(\sec^2 2)$

$$\frac{dy}{dx} = 3\left(\tan(x^4+x)\right)^2 \cdot \left(\frac{d}{dx}\left(\tan(x^4+x)\right)\right)$$

$$= 3\left(\tan(x^4+x)\right)^2 \cdot \left(\sec^2(x^4+x)\right) \cdot (4x^3+1)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 3\left(\tan(2)\right)^2 \left(\sec^2(2)\right) \cdot (5)$$