Solutions - Exam 2 - Math 162
Fall 2004

1. If $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx = \int_{-\infty}^{0} \frac{e^x dx}{1+e^{2x}} + \int_{0}^{\infty} \frac{e^x dx}{1+e^{2x}} = I + II$ then

a) $I = \frac{\pi}{2}$, $II = \frac{\pi}{4}$

b) $I = \frac{\pi}{4}$, $II = \frac{\pi}{4}$

c) $I = \frac{\pi}{4}$, $II = \frac{\pi}{2}$

d) $I = \infty$, $II = \infty$

e) $I = \frac{\pi}{2}$, $II = \infty$

$u = e^x$, $du = e^x dx$: $\displaystyle\int \frac{e^x dx}{1+e^{2x}} = \int \frac{du}{1+u^2}$

$= \tan^{-1}(u) + c = \tan^{-1}(e^x) + c$

So $\displaystyle\int_{-\infty}^{0} \frac{e^x dx}{1+e^{2x}} = \lim_{a \to -\infty} \left[\tan^{-1}(1) - \tan^{-1}(e^a)\right]$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4} = I$

$II = \displaystyle\int_0^{\infty} \frac{e^x dx}{1+e^{2x}} = \lim_{a \to \infty} \left[\tan^{-1}(e^a) - \tan^{-1}(1)\right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(b)

2. The approximation to $\displaystyle\int_0^{\pi} \sin x\,dx$ using the midpoint rule with $n = 3$ is

$0 \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \pi$

a) $\frac{\pi}{3}$

b) $\frac{1}{2}$

c) π

d) $\frac{2\pi}{3}$

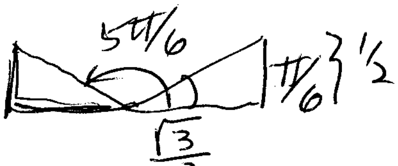
e) 1

$\Delta x = \frac{\pi}{3}$

approx. is

$\frac{\pi}{3}\left(\sin\frac{\pi}{6} + \sin\frac{\pi}{2} + \sin\frac{5\pi}{6}\right)$

$= \frac{\pi}{3}\left(\frac{1}{2} + 1 + \frac{1}{2}\right) = \frac{2\pi}{3}$



(d)

3. The length of the curve $y = \dfrac{e^x + e^{-x}}{2}$ for $0 \le x \le 1$ is given by $L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

a) $\int_0^1 \dfrac{(e^x + e^{-x})}{2} dx$

b) $\int_0^1 \sqrt{1 + \dfrac{(e^{2x} - e^{-2x})}{4}}\, dx$

c) $\int_0^1 \dfrac{\sqrt{1 + e^{2x} + e^{-2x}}}{2}\, dx$

d) $\int_0^1 \dfrac{\sqrt{e^{2x} + e^{-2x}}}{2}\, dx$

e) $\int_0^1 \dfrac{\sqrt{e^{2x} - e^{-2x}}}{2}\, dx$

$\dfrac{dy}{dx} = \dfrac{e^x - e^{-x}}{2}$, so $\boxed{1 + \left(\frac{dy}{dx}\right)^2} =$

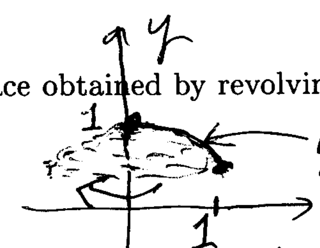
$1 + \dfrac{e^{2x} - 2 + e^{-2x}}{4} = \dfrac{e^{2x} + 2 + e^{-2x}}{4}$

$\boxed{= \dfrac{(e^x + e^{-x})^2}{4}}$.

So $L = \int_0^1 \dfrac{(e^x + e^{-x})}{2}\, dx$.

a

4. Find the area of the surface obtained by revolving $y = 1 - \dfrac{x^2}{2}$, $0 \le x \le 1$ about the y–axis.



a) $\frac{\pi}{4}$

b) $\dfrac{2\pi}{5}$

c) $\dfrac{\pi}{3}(2^{3/2} - 2)$

d) $\dfrac{\pi}{5}(2^{7/2} - 2)$

e) $\dfrac{\pi}{3}(2^{5/2} - 2)$

$\text{area} = \int_0^1 2\pi \cdot \text{radius}\, ds$

$= \int_0^1 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^1 2\pi x \sqrt{1 + x^2}\, dx$

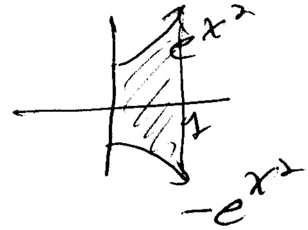
$= \dfrac{2\pi}{3} (1 + x^2)^{3/2} \Big|_0^1$

$= \dfrac{2\pi}{3}(2^{3/2} - 1) = \dfrac{\pi}{3}(2^{5/2} - 2)$

e

5. Find the moment about the y–axis of the region between $y = -e^{x^2}$ and $y = e^{x^2}$ for $0 \leq x \leq 1$.

$$M_y = \int_a^b x[f(x) - g(x)]\,dx$$



$$= \int_0^1 x[e^{x^2} + e^{x^2}]\,dx$$

a) 0

b) 1

c) $e - 1$

d) $e^2 - 1$

e) $\frac{(e-1)}{2}$

$u = x^2$

$du = 2x\,dx$

$$= \int_0^1 e^u\,du = e^u\Big|_0^1 = e - 1$$

c

6. $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{3^n}$ equals

$$= 2 \cdot \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^n} = 2\left[\left(\frac{-2}{3}\right) + \left(\frac{-2}{3}\right)^2 + \left(\frac{-2}{3}\right)^3 + \cdots\right]$$

$$= -\frac{4}{3}\left[1 + \left(\frac{-2}{3}\right) + \left(\frac{-2}{3}\right)^2 + \cdots\right]$$

$$= -\frac{4}{3} \cdot \frac{1}{1 - \left(\frac{-2}{3}\right)}$$

$$= \left(\frac{-4}{3}\right) \cdot \frac{3}{5} = \frac{-4}{5}$$

a) 0

b) $-\frac{2}{5}$

c) $\frac{2}{5}$

d) $\frac{4}{5}$

e) $-\frac{4}{5}$

e