MA 162 EXAM **September 11, 2001**

Name _____ Student ID number _____

Lecturer _____ Recitation Instructor _____

Instructions:

1. This package contains 13 problems, each worth 8 points, for a total of 104 points.
2. Please supply all information requested above and on the mark–sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly.
4. No books, notes, or calculator, please.

1. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors in the space, which among the following statements is/are true:

   I. $\mathbf{a} + \mathbf{b}$ is a vector

   II. $\mathbf{a} \times \mathbf{b}$ is a vector.

   III. $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is a vector

   A. Only I
   B. Only II
   C. Only III
   D. Only I and II
   E. Only I and III

2. A pilot is steering his plane in the direction N30°E, i.e., 30° east of due north, at an airspeed (speed in still air) of 60 miles/hour. If the wind is blowing from west to east at a speed of 20 miles/hour, what is the speed of the plane relative to the ground, in miles/hour?

   A. $\sqrt{2800}$
   B. $60 + 10\sqrt{3}$
   C. 70
   D. $\sqrt{5200}$
   E. $60 - 10\sqrt{3}$

3. The angle between $\mathbf{v} = \langle 2, 2, 1 \rangle$ and $\mathbf{w} = \langle 1, 0, 1 \rangle$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\cos^{-1} \frac{1}{3\sqrt{2}}$

D. $\cos^{-1} \frac{1}{3}$

E. $\frac{\pi}{4}$

4. Find the area of the parallelogram with vertices $(1, 0, 1)$, $(2, 2, 1)$, $(3, 4, 3)$, $(2, 2, 3)$

A. $4\sqrt{5}$

B. $3\sqrt{5}$

C. $2\sqrt{5}$

D. $2\sqrt{6}$

E. $4\sqrt{6}$

5. Find a vector of length 2 that is orthogonal to both $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

A. $\frac{2}{\sqrt{5}}\,\mathbf{i} + \frac{1}{\sqrt{5}}\,\mathbf{j}$

B. $-\frac{4}{\sqrt{5}}\,\mathbf{i} + \frac{2}{\sqrt{5}}\,\mathbf{j}$

C. $\frac{4}{\sqrt{5}}\,\mathbf{i} + \frac{2}{\sqrt{5}}\,\mathbf{j}$

D. $\sqrt{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{k}$

E. $\sqrt{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{k}$

6. The volume of the parallelepiped determined by the vectors $\langle 1, 2, -1\rangle$, $\langle 2, 1, 1\rangle$, and $\langle 0, 1, 1\rangle$ is

A. 2

B. 4

C. 5

D. 6

E. 7

7. Find the vector projection of the vector $\langle 1, 1, 2\rangle$ onto the vector $\langle 2, 1, 2\rangle$.

A. $\langle \frac{14}{3}, \frac{7}{3}, \frac{14}{3}\rangle$

B. $\langle \frac{14}{9}, \frac{7}{9}, \frac{14}{9}\rangle$

C. $\langle \frac{14}{3}, \frac{5}{3}, \frac{10}{3}\rangle$

D. $\langle \frac{10}{9}, \frac{5}{9}, \frac{10}{9}\rangle$

E. $\langle \frac{10}{3}, \frac{5}{3}, \frac{10}{3}\rangle$

8. The area of the shaded region is

A. $13\frac{1}{3}$

B. 12

C. $4\sqrt{6} - 1$

D. $4\sqrt{6} - \frac{2\sqrt{2}}{\sqrt{3}}$

E. $3\frac{5}{6}$

9. Which of these formulas give(s) the area of the region between the x axis and the curve
$y = x(x^2 - 1),\ -1 \leq x \leq 1$?

I. $\int_{-1}^{1} x(x^2-1)dx$ II. $\int_{-1}^{1} |x(x^2-1)|dx$ III. $\int_{-1}^{0} x(x^2-1)dx - \int_{0}^{1} x(x^2-1)dx$

A. Only I
B. Only II
C. Only I and II
D. Only II and III
E. All three

10. A solid is placed in a rectangular coordinate system. Its base is the region in the xy plane between the x axis and the curve $y = \sin x,\ 0 \leq x \leq \pi$, and its cross sections perpendicular to the x axis are semicircles. The volume of this solid is then given by the integral

A. $\frac{\pi}{4} \int_{0}^{\pi} \sin^2 x dx$
B. $\frac{\pi}{8} \int_{0}^{\pi} \sin^2 x dx$
C. $\frac{\pi}{2} \int_{0}^{\pi} \cos x dx$
D. $\frac{\pi}{4} \int_{0}^{1} \sin x dx$
E. $\frac{\pi}{2} \int_{0}^{1} \cos^2 x dx$

11. The triangle in the xy plane bounded by the lines $y = x$, $y = \frac{x}{2}$, $y = 1$ is rotated about the y axis. The volume of the solid thus generated is

A. π

B. 2π

C. 3π

D. 6π

E. 8π

12. It takes a force of 20N to keep a spring stretched 20 cm = 0.2 m beyond its natural length. How much work (in J= Newton $\times$ meter) is needed to stretch it by an additional 20 cm?

A. 2

B. 20

C. 12

D. 36

E. 6

13. On a certain night, the temperature at t hours past midnight is $\left(\frac{t^2}{2} - 2t + 50\right)$ °F, $0 \leq t \leq 6$. Find the average temperature, in °F, between midnight and 6 a.m.

A. 48

B. 50

C. 52

D. 54

E. 56