NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(8) 1. Find $f''(x)$ if $f(x) = (1-x^2)^{3/2}$.

or 4

$$f'(x) = \frac{3}{2}(1-x^2)^{1/2}(-2x) = -3x(1-x^2)^{1/2}$$

$$f''(x) = -3x\,\frac{1}{2}(1-x^2)^{-1/2}(-2x) - 3(1-x^2)^{1/2}$$

$$= 3x^2(1-x^2)^{-1/2} - 3(1-x^2)^{1/2}$$

or 4

$$\boxed{3x^2(1-x^2)^{-1/2} - 3(1-x^2)^{1/2}}$$

8

(10) 2. Find an equation of the line tangent to the graph of the equation $xy^2 = 18$ at the point $(2, -3)$.

$$xy^2 = 18$$

$$x\,2y\frac{dy}{dx} + y^2 = 0 \quad 4$$

At $(2,-3)$: $2\cdot 2\cdot(-3)\dfrac{dy}{dx} + (-3)^2 = 0$

$$\frac{dy}{dx} = \frac{9}{12} = \frac{3}{4} \quad 3$$

3 ok if consistent with wrong slope

$$\boxed{y+3 = \frac{3}{4}(x-2)}$$

10

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(12) 3. Boyle's Law states that if the temperature of a gas remains constant, then the pressure p and the volume V of the gas satisfy the equation $pV = c$, where c is a constant. If the volume is decreasing at the rate of 10 cubic centimeters per second, how fast is the pressure increasing when the pressure is 100 pounds per square centimeter and the volume is 20 cubic centimeters?

$pV = c \qquad \frac{dV}{dt} = -10$

$p\frac{dV}{dt} + \frac{dp}{dt}V = 0$ 6

When $p = 100$ and $V = 20$;

$100(-10) + \frac{dp}{dt} \cdot 20 = 0$ 4

$\frac{dp}{dt} = 50$

2

$\boxed{50 \text{ lbs/cm}^2\text{/sec}}$ $\boxed{12}$

(8) 4. Approximate $\sqrt{101}$ using a linear approximation.

4 → $f(a+h) \approx f(a) + f'(a)h$ for small h

$f(x) = \sqrt{x} \qquad a = 100 \qquad h = 1$

$f'(x) = \frac{1}{2\sqrt{x}} \qquad f(100) = 10 \qquad f'(100) = \frac{1}{2 \cdot 10} = \frac{1}{20}$

$\sqrt{101} \approx \sqrt{100} + \frac{1}{20} \cdot 1 = \underbrace{10 + \frac{1}{20}}_{} = 10.05$

−1pt for minor numerical error or 4 → $\boxed{10.05}$ $\boxed{8}$

(10) 5. Find the maximum and minimum values of the function $f(x) = e^x - e^{2x}$ on the interval $[0, 1]$ and the points in $[0, 1]$ where these values occur.

$f'(x) = e^x - 2e^{2x}$ 2

$f'(x) = 0: \quad e^x - 2e^{2x} = 0$

$e^x(1 - 2e^x) = 0$

$e^x = \frac{1}{2} \quad x = -\ln 2$ ← not in $(0,1)$

$f(0) = 0$

$f(1) = e - e^2$

3 1

maximum value
$f(0) = 0$

$\boxed{10}$

minimum value
$f(1) = e - e^2$

3 1

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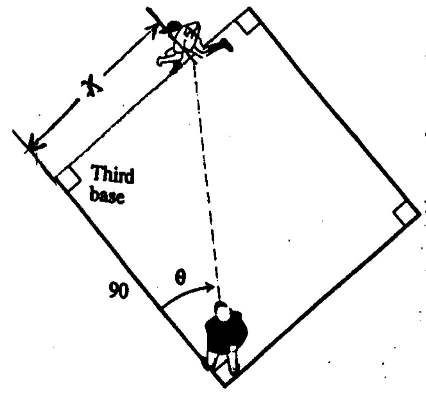
(14) 6. A baseball diamond is a square with sides 90 feet long. Suppose a baseball player is advancing from second base to third base at the rate of 24 feet per second, and an umpire is standing at home plate. Let θ be the angle between the third baseline and the line of sight from the umpire to the runner. How fast is θ changing when the runner is 30 feet from third base?



$\tan\theta = \frac{x}{90}$ (4) $\frac{dx}{dt} = -24$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{90}\frac{dx}{dt}$ (4)

When $x = 30$:

$\sec\theta = \frac{\sqrt{90^2+30^2}}{90}$

$\sec^2\theta = \frac{90^2+30^2}{90^2} = 1 + \frac{1}{9} = \frac{10}{9}$ (3)

$\frac{10}{9}\frac{d\theta}{dt} = \frac{1}{90}(-24)$

$\frac{d\theta}{dt} = -\frac{24}{100}$

$= -\frac{6}{25}$

or (3)

$\boxed{\frac{6}{25} \text{ rads/sec}}$ 14

(6) 7. Find all critical numbers of $f(x) = x + \sin x$ in the interval $(-2\pi, 2\pi)$.

$f'(x) = 1 + \cos x$

$f'(x) = 0$: $1 + \cos x = 0$

$\cos x = -1$

$x = -\pi, \pi$

−3 pts for each additional number

(3) (3)

$\boxed{-\pi, \pi}$ 6

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(10) 8. Find all intervals on which the function $f(x) = x^4 - 2x^3 + 1$ is increasing and all intervals on which it is decreasing.

$f'(x) = 4x^3 - 6x^2$ (2)

$f'(x) = 2x^2(2x-3)$

$f'(x) = 0: \quad 2x^2(2x-3) = 0 \rightarrow x = 0 \quad x = \frac{3}{2}$ (1) (1)

$f'(x)$: $- - - - - \,|\, - - - - \,|\, + + + +$ at 0 and $\frac{3}{2}$

(3) f is increasing on $[\frac{3}{2}, \infty)$

(3) f is decreasing on $(-\infty, \frac{3}{2}]$

$(-\infty, 0]$ and $[0, \frac{3}{2}]$ OK

$\boxed{10}$

(10) 9. Determine the function f satisfying the conditions

$$f''(x) = \cos x, \qquad f'(\tfrac{\pi}{2}) = 2, \qquad f(0) = 4$$

$f'(x) = \sin x + C_1$

$x = \frac{\pi}{2}: \quad 2 = \sin\frac{\pi}{2} + C_1 \longrightarrow C_1 = 1$

$f'(x) = \sin x + 1$

$f(x) = -\cos x + x + C_2$

$x = 0: \quad 4 = -\cos 0 + 0 + C_2 \rightarrow C_2 = 5$

(4) (3) (3)

$f(x) = -\cos x + x + 5$

$\boxed{10}$

(12) 10. Suppose you have a cache of radium whose half-life is approximately 1590 years. How long would you have to wait for one tenth of it to disappear? You may leave your answer in terms of logarithms.

$f(t) = f(0)e^{kt}$ (2)

half-life 1590 yrs $\rightarrow \frac{1}{2} f(0) = f(0)e^{k(1590)}$

$-\ln 2 = k(1590)$

$k = -\frac{\ln 2}{1590}$ (5)

$f(t) = f(0)e^{-\frac{\ln 2}{1590}t}$

$\frac{9}{10} f(0) = f(0)e^{-\frac{\ln 2}{1590}t}$ (3)

$\ln(\frac{9}{10}) = -\frac{\ln 2}{1590}t$

$t = -1590\,\frac{\ln(\frac{9}{10})}{\ln 2}$

(2)

$-1590\,\frac{\ln(\frac{9}{10})}{\ln 2}$

$\boxed{12}$